Name: ______________________________

Project 5: Kepler's Laws and Lazlo's Comet

In this project you will determine whether or not the planets in your new solar system obey Kepler's Third (Harmonic) Law. You will also apply Kepler's Laws of Planetary Motion to understand and predict the behavior of a comet that has just appeared in your solar system. You may even be able to determine the mass of your new Sun (Barnard's Star). *This project is due in class on Tuesday, 26 November, 2013.*

Kepler's Third Law in Your Solar System

The first thing you will do for this project is to determine whether or not your solar system follows Kepler's Third Law. Recall that Kepler's Third Law states that $T^2 = a^3$ where T is the period of a planet's orbit in years (where a year is the orbital period of your home planet) and a is the semi-major axis of the planet's elliptical orbit in AU (where 1 AU is the semi-major axis of your home planet's orbit). Note that in your system the planets all orbit in circles with the Sun at the center. A circle is just an ellipse with an eccentricity of zero (both foci are located at the center), so the semi-major axis of your orbits is simply the radius.

1. Complete the table below using the results from the Copernican model of your solar system. [20 points]

Planet Name	T **(days)**	T **(years)**	a **(AU)**	T^2 **(yrs^2)**	a^3 **(AU^3)**

2. Do the results in the table above indicate that your solar system follows Kepler's Third Law? Explain your answer. [2 points]

Lazlo's Comet

Your group of colonists has established an outpost on one of the more distant planets in your new solar system. An astronomer at that outpost, one Dr. Cedric Lazlo, detected a comet and worked out the details of its orbit. A diagram depicting the comet's orbit and showing the location of your system's Sun, as well as a circle indicating the background stars along the ecliptic plane, is attached to this handout. The comet moves counterclockwise along the elliptical orbit in the diagram. You would like to make use of this diagram to understand and predict the motion of the comet and, in combination with measurements of the comet's parallax, to determine the radius of your home planet's orbit in standard units (like miles). But first, you must determine the location of your home planet's orbit on this diagram.

To that end you make an observation of the comet on the day the comet was predicted by Dr. Lazlo to reach its first quadrant point (marked $Q1$ on the diagram). This day will be referred to as Day 0. On Day 0 you observe the comet to be at ecliptic longitude ♐ 20°. On the same day you observe the Sun to be at ecliptic longitude ♏ 10°. Using this information, and the diagram you were given, you must now determine the orbit of your home planet, as well as many details about the comet's orbital motion.

1. Before you start you need to gather some data from previous projects. Record below the radius R of your home planet in miles from Project 1, as well as the sidereal day T_d for your home planet in hours (recall that the sidereal day is also the rotational period of your home planet). [2 points]

   $R =$ __________________________mi, $T_d =$ __________________________hours

2. Use Kepler's triangulation method to mark the location of your home planet on Day 0 in your diagram. Make sure to clearly draw any lines or other geometrical figures that you used to find this location. Label this point t_0. [6 points]

3. Draw your home planet's circular, Sun-centered orbit on your diagram. Use a compass and draw it as carefully is you can. [3 points]

4. Use a ruler to measure the radius R_H of this circle, in centimeters. Record your answer below. Note that this distance corresponds to one Astronomical Unit in your diagram. [1 point]

   $R_H =$ __________________________cm

5. Measure the distance d_c on your diagram from your home planet to the comet on Day 0, in centimeters. Record your answer below. [1 point]

   $d_c =$ __________________________cm

6. Convert the distance d_c into Astronomical Units. Show your work and your result below. [2 points]

7. On Day 0, you and a colleague make simultaneous measurements of the apparent location of the comet from opposite points on the equator on your home planet. After comparing the two measurements you find a difference of $30''$ in the comet's position taken from the two locations. Therefore, the parallax angle for the comet observed from opposite sides of your home planet is $15''$. What is the baseline b for this parallax measurement, in miles? Record your answer below. [1 point]

$b =$ __________________________ mi

8. How far away from your home planet is the comet, in miles, on Day 0? Show your work and give your answer below. [2 points]

9. Compare the distance you just found to the distance d_c you found above. How many miles are in one Astronomical Unit for your solar system? Show your work and give your answer below. [2 points]

10. Measure the aphelion and perihelion distances for your comet's orbit on your diagram. Record your results, in centimeters, below. [2 points]

$d_a =$ __________________________ cm, $d_p =$ __________________________ cm

11. Convert d_a and d_p to Astronomical Units. Record your results below. [2 points]

$d_a =$ __________________________ AU, $d_p =$ __________________________ AU

12. Calculate the eccentricity e of the comet's orbit. Show your work and give your answer below. [2 points]

13. Determine the semi-major axis a of the comet's orbit. Show your work and give your answer below. [2 points]

14. Use Kepler's Third Law to determine the period T of this comet's orbit, in years. Show your work and give your answer below. [2 points]

15. Determine the period of the comet's orbit in days. Recall that a year is defined as the orbital period of your home planet. Show your work and give your answer below. [2 points]

16. Use Kepler's Second Law to determine the approximate day on which the comet will reach the perihelion of its orbit. Recall that the comet was at first quadrant on Day 0. Give your answer below and explain how you found it. [2 points]

17. Now determine the approximate day on which the comet will reach the third quadrant of its orbit. Give your answer below. [2 points]

18. Determine the approximate day on which the comet will reach the aphelion of its orbit. Give your answer below. [2 points]

19. Through how many degrees of its orbit will your home planet have moved between Day 0 and the day the comet reaches *perihelion*? (Note that if you answered 360° that would mean that your planet has completed one full orbit in that time.) Show your work and give your answer below. [2 points]

20. Mark the approximate location of your home planet on your diagram for the day when the comet reaches perihelion. Label this point t_P. [2 points]

21. Determine the approximate ecliptic longitude of the *comet*, as seen from your home planet, on the day the comet reaches perihelion. Give your answer below. On your diagram you should show any lines or other geometrical objects that you used to determine this result. [4 points]

22. Determine the approximate ecliptic longitude of the *Sun*, as seen from your home planet, on the day the comet reaches perihelion. Give your answer below. On your diagram you should show any lines or other geometrical objects that you used to determine this result. [4 points]

The Mass of Barnard's Star

Using the information you have gathered it is possible to determine the mass of Barnard's Star in solar masses (where a solar mass refers to the mass of Earth's Sun, or Sol).[1] You can use the same procedure that we used to determine the masses of Jupiter and the Earth. However, in order to do so you must convert times into Earth years and distances into Earth Astronomical Units. Recall that not only does the year of your new home planet have a different number of days than Earth's year, but your day has a different number of hours than Earth's day (an hour, however, is the same for both systems). You may need to look up some information about Earth's Astronomical Unit. To receive full credit for your answer, you must show all of your work in the space below. Good luck! [3 bonus points]

[1] DO NOT TRY TO LOOK THIS UP! You might be able to find the actual mass of Barnard's Star online, but for this project you must determine the mass of the star from the data you collected on the fictitious planets in your new solar system. Because that data is made up, so too is the mass of the star. Your value will most likely be very different from the true mass of the real Barnard's Star.