AST 120 Activity 19
Kepler's Attacks on Mars: The First Law

Name	Full	Partial	None

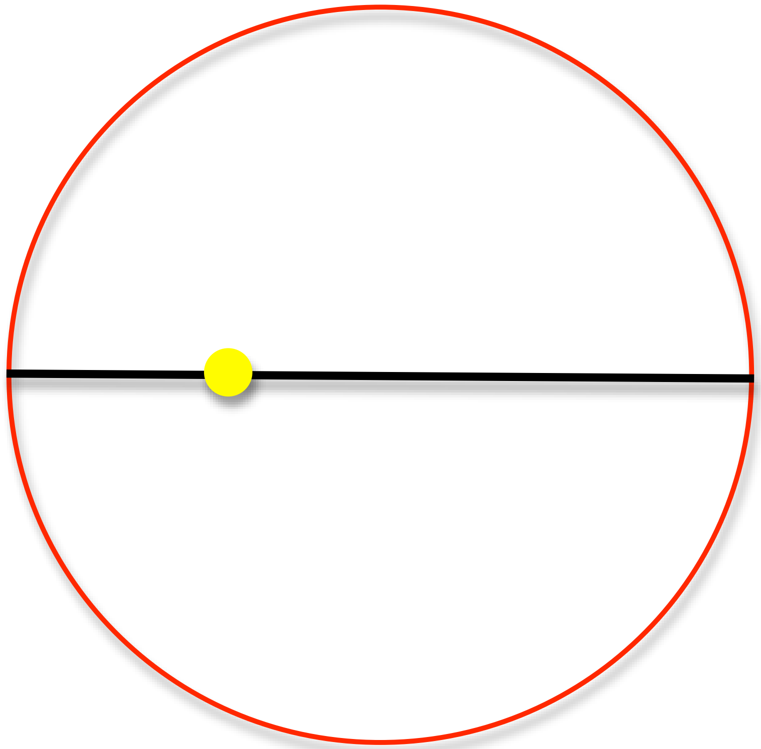
At this point in the *Astronomia Nova* Kepler has established a new orbit for Earth, and with it a new law for describing the motion of a planet on its orbit (what we now call Kepler's Second Law, which states that the line from the Sun to the planet sweeps out equal areas in equal times).[1] He is ready to take another crack at finding an orbit for Mars. Recall that he had built an orbit for Mars using an eccentric/equant construction, but that had shown errors on the order of 8' of arc. His first step now is to discard the equant and instead use his new Law of Areas to describe the motion of Mars on its eccentric circular orbit. To get an idea of how this orbit works run the `KeplerAstronomiaNovaOrbits` program. In the Display Options menu, deselect the oval and elliptical orbits.

1. The simulation shows Kepler's circular orbit with greatly exaggerated eccentricity. Play the simulation and watch the motion of the planet on its orbit. The lower window shows the motion of the planet relative to the background stars *as seen from the Sun* (NOT as seen from the Earth). Does the planet maintain a uniform speed?

2. Let's establish some important vocabulary. When the planet is closest to the Sun we say it is at *perihelion*. When it is farthest from the Sun we say it is at *aphelion*. If we measure the angle between the line connecting the Sun to the aphelion point and the line connecting the Sun to the planet's location, then we say the planet is at a *quadrant* if that angle is 90° (first quadrant) or 270° (third quadrant). Similarly, we refer to angles of 45°, 135°, 225°, and 315° as *octants* (first, third, fifth, and seventh, respectively). Explain why it makes sense to call an angle of 225° the fifth octant.

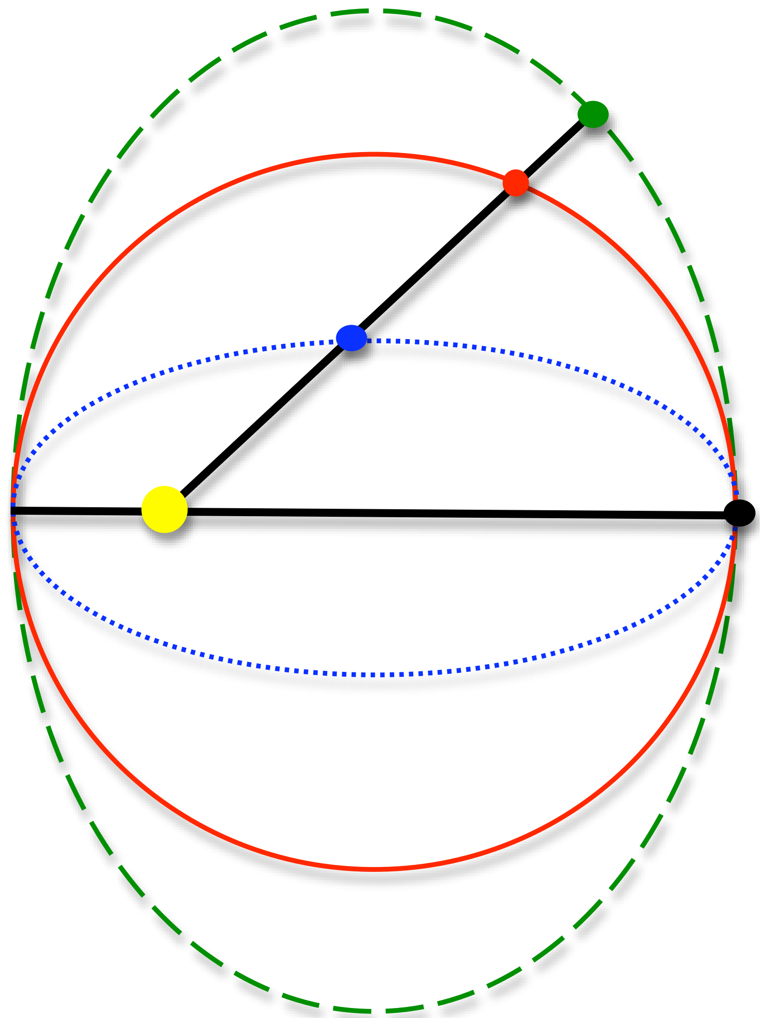
[1]Kepler never actually took the Law of Areas as a fundamental law. He always thought of it as an approximation to his velocity law ($v \propto 1/r$). In fact, he never referred to it as a "Law" at all.

3. In the diagram of Kepler's Mars orbit shown below, mark and label the perihelion point (`p`), the aphelion point (`a`), the two quadrants (`q1` and `q3`), and the four octants (`o1`, `o3`, `o5`, and `o7`). [Hint: make sure you are measuring the angles from the Sun, not from the center of the circle.]



4. In Kepler's new circular orbit Mars is moving fastest when it is at ______________.
   (a) perihelion
   (b) aphelion
   (c) first quadrant
   (d) third quadrant
5. In Kepler's new circular orbit Mars is moving slowest when it is at ______________.
   (a) perihelion
   (b) aphelion
   (c) first quadrant
   (d) third quadrant
6. Do your answers to the two previous questions fit with the Law of Areas? Briefly explain. .

7. When Kepler compared his new orbit to Tycho's data he found the same problem that he had found with his eccentric/equant orbit, namely errors on the order of 8'. Specifically, he found that his new orbit predicted a time between aphelion and first octant that was too short. There were two possible ways to fix this: either change the law that governs how fast the planet moves on its circle (the Law of Areas), or change the shape of the orbit. Kepler used triangulation to determine the location of Mars at several points on its orbit and became convinced that the orbit was non-circular. But he couldn't determine the shape. Consider the diagram shown below, which shows three possible orbits. The solid line is a circular orbit. All three orbits match at aphelion and perihelion.



Imagine three planets that all start at the aphelion point. The planets then move along each of the three orbits until they reach the location shown in the diagram (roughly the first octant). Recall that the Law of Areas tells us that the time it takes for a planet to move a certain distance along its orbit (as a fraction of the orbital period) is given by the area swept out by the Sun-planet line (as a fraction of the total area inside the orbit). Shade the area swept out by the Sun-planet line between aphelion and first **octant** if the planet is moving along the dotted path. Approximately what percent of the total area *inside the oval path* is shaded? The orbital period of Mars is 687 days. Use Kepler's Law of Areas to determine the time it takes for the planet to move from aphelion to the first octant if it moves along this oval path.

8. Now go back and shade the portion swept out if the planet moves from aphelion to first octant along the solid (circular) orbit. [Note that you have already shaded part of this in the previous question.] Approximately what percent of the total area *inside the solid circle* have you shaded? Use Kepler's Law of Areas to determine the time it takes for the planet to move from aphelion to the first octant if it moves along this circular path.

9. Now go back and shade the portion swept out if the planet moves from aphelion to first octant along the dashed orbit. [Note that you have already shaded part of this in the previous two questions.] Approximately what percent of the total area *inside the dashed orbit* have you shaded? Use Kepler's Law of Areas to determine the time it takes for the planet to move from aphelion to the first octant if it moves along this circular path.

10. Along which of these three orbits will the planet take the shortest amount of time to go from aphelion to first octant?

11. Along which of these three orbits will the planet take the greatest amount of time to go from aphelion to first octant? [Note; double-check your last few answers with Dr T before moving on.]

12. To solve the 8' error does Kepler need an orbit *contained within* his circular orbit, or an orbit that *lies outside* his circular orbit?

13. In the Display Options menu select Show Oval Orbit to see Kepler's next attempt at constructing an orbit for Mars. Kepler knew that one way to construct a non-circular orbit was to add an epicycle onto a circular orbit. But he wasn't entirely comfortable with this idea since he didn't see how a physical force could lead to epicyclic motion. Still, it was worth a shot. He built an orbit with a deferent centered on the Sun (shown in green) and an epicycle (shown in blue). He was still committed to his Law of Areas but wasn't sure how to make it work with this setup. He realized he could approximate the Law of Areas by making the epicycle rotate uniformly but with the center of the epicycle moving non-uniformly on the deferent *the same way that Mars had moved on the previous circular orbit.*[2] The simulation shows Kepler's previous circular orbit in red as well as the new deferent-epicycle orbit in cyan (with grossly exaggerated eccentricity). The motion of the planet along the epicycle of the new orbit is in a direction ______________ that of the motion of the center of the epicycle along the deferent.

    (a) opposite
    (b) the same as

[2]This only approximates the Law of Areas if the eccentricity is small. In the simulation the eccentricity has been grossly exaggerated to illustrate the shape of the resulting orbit - so the Law of Areas wouldn't actually hold in this case.

14. Near the first octant, the planet on the new oval orbit is ______________ the planet on the old circular orbit. Near the third octant the planet on the new oval orbit is ______________ the planet on the old circular orbit.

    (a) ahead . . . ahead
    (b) behind . . . behind
    (c) ahead . . . behind
    (d) behind . . . ahead

15. Describe the shape of the resulting orbit? Is it enclosed within the circular orbit or does it lie outside of the circular orbit? Is it symmetric? What common object does it remind you of?

16. Does it take the new orbit longer to go from aphelion to the first octant, as desired to correct the 8' error?

17. Unfortunately, Kepler found that this *ovoid* orbit went too far in the other direction. The time from aphelion to first octant was too long! In fact, this orbit also gave 8' errors at the octants, but in the opposite direction from the errors of the circular orbit. He knew he needed something in between. While pondering this problem he noticed an apparent numerical coincidence (mentioned in the text) that eventually led him to conclude that he needed to use an *ellipse*. So he constructed an elliptical orbit that satisfied the Law of Areas and it matched up almost perfectly with Tycho's observations of Mars! In the Display Options menu select Show Elliptical Orbit to see how Kepler's ellipse compares to the circular and ovoid orbits discussed above. The elliptical orbit is shown in magenta. Watch the simulation. The position of Mars on the elliptical orbit is always ______________ the positions of Mars on the circular and ovoid orbits.

    (a) ahead of
    (b) in between
    (c) behind

18. Is the Sun at the center of the elliptical orbit? If not, is it displaced along the long axis of the ellipse or along the short axis of the ellipse (or in some other direction)?

Kepler realized that the Sun must actually reside at a *focus* of Mars' elliptical orbit (we'll study ellipses in greater detail in the next lab). His victory over Mars was complete. He had broken free from the millennium-old dogma of circular orbits. He had found that Mars moved on an ellipse, with the Sun at one focus and in such a way that the Sun-Mars line would sweep out equal areas in equal times. Kepler would go on to apply this principle to constructing the orbits of all the other planets (including the Earth). This was the most important breakthrough in planetary astronomy at the time, and nothing since has topped it. Except for some minor modifications (due to gravitational interactions between planets and the effects of General Relativity) we still use Kepler's two laws for describing the motion of the planets.