# Shadows and Gnomons

Laboratory 2

Astronomy 120. The Copernican Revolution

| **Name** | **Full** | **Partial** | **None** |
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## INTRODUCTION

One of the first astronomical measuring instruments was the *gnomon*, which in its simplest form is a straight stick stuck vertically into the ground. The gnomon is used to track both the daily and the longer term movement of the Sun in the sky. (Couple the gnomon with a graduated scale and you have a sundial.) Even though the gnomon has ancient roots, it can be used to outline the way modern scientists organize and analyze observed data – typically from *table* to *graph* to *equation*.

To understand the genesis of the gnomon, consider what general features of the Sun's movement ancient observers would have noticed:

- Each day, the Sun rises somewhere in the eastern half of the sky and sets somewhere in the western half.
- Shadows are longer during the early mornings and late afternoons when the Sun is low in the sky; they are shorter around midday when the Sun is high.
- The Sun climbs higher in the sky during summer than during winter.
- Summer days have more hours of daylight than winter days; that is, the Sun rises earlier and sets later during summer.

The gnomon was probably introduced in an effort to quantify these general observations, that is, to measure the Sun's movement in a consistent and reliable way. Then the Sun could serve both as a daily clock and as a way to track the passage of the seasons in order to decide such essential matters as when to plant crops, when to store food, when to prepare shelter. As you can see, there were very practical and even life-sustaining reasons for early civilizations to pursue the study of astronomy.

## PART 1: GETTING FAMILIAR WITH THE GNOMON

During the course of the day, the altitude and azimuth of the Sun change. The change in the Sun's altitude causes the length of the gnomon's shadow to change. The change in the Sun's azimuth causes the direction of the gnomon's shadow to change. To investigate exactly how the shadow changes, run the EJS Gnomon program. This program simulates the shadow of a gnomon as the Sun moves through the sky. You can input any latitude and any day of the year (day 0 corresponds to the vernal equinox, March 21) and see how the shadow of a gnomon at that latitude would look during the course of that day.

1. For each combination of latitude and day, determine the general direction (i.e. NW) of the gnomon's shadow and the general direction of the Sun just after sunrise, at *local noon*[1], and just before sunset. You should also pay attention to how the length of the shadow changes. When you type new values into the simulation, make sure to hit enter after you finish typing. If you haven't hit enter the value will be highlighted in yellow. The simulation won't use the new value unless you hit enter.

| Latitude | Day | Sunrise Shadow | Sunrise Sun | Noon Shadow | Noon Sun | Sunset Shadow | Sunset Sun |
|---|---|---|---|---|---|---|---|
| 35° | 0 (♈) | | | | | | |
| 35° | 92 (SS) | | | | | | |
| 35° | 185 (♎) | | | | | | |
| 35° | 276 (WS) | | | | | | |
| 0° | 0 (♈) | | | | | | |
| 0° | 92 (SS) | | | | | | |
| 0° | 185 (♎) | | | | | | |
| 0° | 276 (WS) | | | | | | |
| -40° | 0 (♈) | | | | | | |
| -40° | 92 (SS) | | | | | | |
| -40° | 185 (♎) | | | | | | |
| -40° | 276 (WS) | | | | | | |

2. Write a sentence describing the relative locations of the Sun and the corresponding gnomons shadow.

[1]This is the time when the Sun is highest in the sky, and therefore it is also the time when the gnomon's shadow is shortest. The simulation has been set so that local noon always occurs when the time is $12^h$. Actually, as we will see later, local noon doesn't occur at quite the same time every day. So the simulation is *actually* set up so that local noon will occur at $12^h$ *on average*. We'll examine this issue in detail later in the lab.

3. Explain why the shadow is longer in the early morning and late afternoon compared to times near midday.

4. At local noon, when the shadow is at its shortest, what direction does the shadow point for an observer at latitude 35° N? What about for an observer at latitude 40° S?

5. If you were lost in the wilds of the Berry College campus, could you use a gnomon to figure out directions? If so, how would you do it?

6. We see that the direction of the noon shadow can change for an observer at latitude 0°. For what *range* of latitudes will the noontime shadow switch directions during the course of the year? Try to figure this out for yourself, then use the simulation to check your answer.

7. Take a look at the simulation for the north pole (latitude 90°) on the summer solstice (Day 92). Describe the behavior of the gnomon's shadow.

8. If you look up from the north pole on the summer solstice how will the Sun appear to move through the sky?

## PART 2: THE GOOD STUFF: EQUATIONS, CALCULATIONS, GRAPHS, PREDICTIONS

In Figure 1 below, notice how the gnomon and its shadow form a right triangle, whose hypotenuse (longest side) is the imaginary line connecting the gnomon's tip to the shadow's endpoint and whose angle $\theta$ is the Sun's altitude in the sky. Simple trigonometry tells us that the tangent (abbreviated "tan") of an angle in a right triangle is equal to the length of the side opposite the angle divided by the length of the side adjacent to the angle.

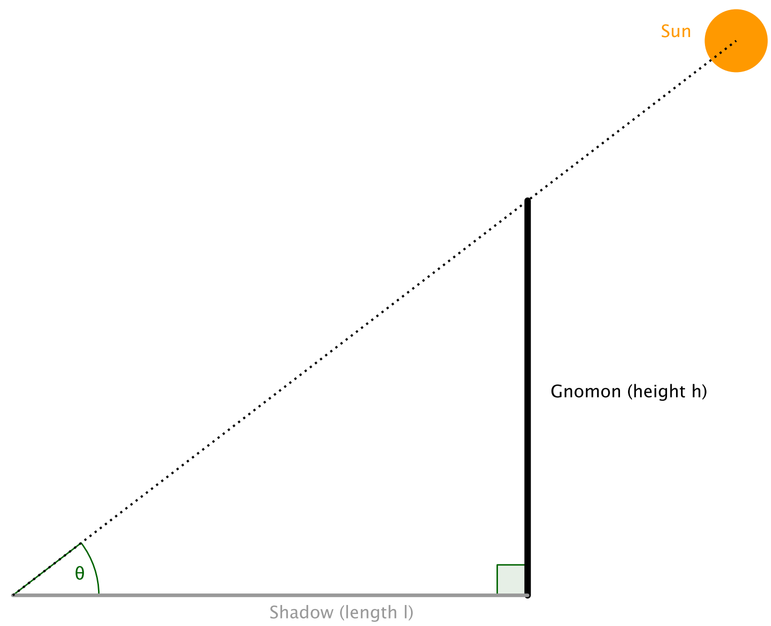


Figure 1. The geometry of the gnomon. (NOT TO SCALE!)

Answer the following questions. Show your work.

1. Using the definition of the tangent, write down a formula relating the gnomon's height $h$, the shadow's length $l$, and the Sun's altitude $\theta$. Write your formula below.

2. How long a shadow would a one-foot gnomon cast if the Sun was 30 degrees above the horizon?

3. Use your formula to compute the altitude of the Sun (in degrees) when a one-foot gnomon casts a half-foot-long shadow. You will need to use the "inverse tangent" key ($\tan^{-1}$) on your calculator. Make sure your calculator is set to accept and display angles in units of degrees, not radians; the calculator display should indicate DEG, not RAD. Ask for assistance if you need it. Show your work and your answer in the space below.

4. Now that you know how to calculate the altitude of the sun at local noon by measuring the length of the noon shadow, let's look at how the sun's local noon altitude changes over the course of a year. Return to the EJS Gnomon program. Set the latitude to 35° and the time to 12 (local noon). Use the Options Menu to Show Altitude Plot. This displays a window that plots the altitude of the sun versus the day of the year. Slowly drag the day slider until you have filled in the plot from day 0 to day 365. In the space below make a sketch of the plot. On the angle scale (the $y$-axis) mark the maximum and minimum values for the sun's altitude, as well as the median (midpoint) value. On the day scale (the $x$-axis) mark the approximate days at which the sun attains its maximum and minimum altitudes, as well as its median altitudes.

5. In your sketch above, mark the locations of the summer solstice (`SE`), winter solstice (`WS`), autumnal equinox (`AE`), and vernal equinox ($VE$).

6. What is the value for the median altitude of the sun as seen from this location? Why is this value significant? (Hint: is it related to the observer's latitude?)

7. What is the difference between the maximum and minimum values and the median value for the sun's altitude? Why is this value significant? (Hint: you've seen this angle before.)

8. Use what you now know about how the sun's altitude at local noon varies over the course of a year to sketch the local noon altitude versus day of the year for an observer at latitude 70° N. DO NOT use the simulation to make this sketch. Do it on your own. Once you have drawn your sketch in the space below, you may check your answer using the simulation.

9. How would the sketch be different for an observer in the southern hemisphere? Think about that, then try to make the sketch for an observer at latitude 50° S. Draw your sketch in the space below. After you have drawn your sketch you may check your answers using the simulation.

## PART 3: THE MEAN SUN VERSUS THE TRUE SUN

In the simulation the time is set so that local noon occurs *on average* at $12^h$. Another way to say this is that the simulation is set up so that the *mean sun* is always highest in the sky at $12^h$. When we refer to the mean sun we are referring to where the sun *would be* if it traveled at a constant rate relative to the stars. But we have already discussed that sometimes the sun travels faster and sometimes it travels slower. The actual location of the sun is referred to as the *true sun*, and this is what is shown in the simulation. Let's use the simulation to explore the difference between the mean sun and the true sun.

1. Set the simulation so that the latitude is 35° and the time is $12^h$. Carefully move the day slider back and forth until you have traced out the the sun's full cycle. Sketch the path traced by the end of the gnomon's shadow at $12^h$ over the course of a full year in the space below. (Note you may want to zoom in by holding Shift, and then clicking and dragging upward in the window.) This shape is known as the *analemma*. Use an arrow to indicate the direction North in your sketch. Mark the location of the shadow tip on each of the following days: Vernal Equinox (March 21), Summer Solstice (June 21), Autumnal Equinox (Sep 21), and Winter Solstice (Dec 21). Note: you can use the Select Day slider to set the simulation to these special days.

2. In the diagram you drew above, the time is always set to $12^h$. We know that the mean sun is due South of the gnomon at this time, so that its shadow points due North. But you have shown that the real shadow (the shadow of the *true* sun) does not always point North at this time. Consider the shadow on the vernal equinox. Does the shadow point due North, or does it point slightly East, or slightly West? What does this tell you about the location of the true Sun relative to the mean Sun? On the vernal equinox, is the true sun lined up with the mean sun, or is it slightly east, or is it slightly west?

3. Now look at the summer solstice. Is the true sun lined up with the mean sun? If not, is it a little bit east or west?

4. During the Spring (from the vernal equinox to the summer solstice) does the true sun move faster or slower than the mean sun, relative to the stars? Remember that the motion of the sun relative to the stars is eastward. During this time period does the true sun move eastward relative to the mean sun (which means it is moving faster), or westward (which means it is moving slower)?

5. Now look at the autumnal equinox. Is the true sun lined up with the mean sun? If not, is it a little bit east or west?

6. During the Summer (from the summer solstice to the autumnal equinox) does the true sun move faster or slower than the mean sun, relative to the stars?

7. Now look at the winter solstice. Is the true sun lined up with the mean sun? If not, is it a little bit east or west?

8. During the Fall (from the autumnal equinox to the winter solstice) does the true sun move faster or slower than the mean sun, relative to the stars?

9. Now look back at where the sun was on the vernal equinox. During the Winter (from the winter solstice to the vernal equinox) does the true sun move faster or slower than the mean sun, relative to the stars?

10. Based on our results for the way the speed of the sun (relative to the stars, or to the mean sun) varies over the course of a year, we might expect different seasons to have different durations. If the sun is moving faster than average during a certain season we would expect that season to be shorter (because it moves along that portion of the ecliptic quickly). If the sun is moving slower than average during a certain season we would expect that season to be longer. Which seasons would we expect to be shorter, which would we expect to be longer, and which would have about average length?

11. Why are there fewer days in February than in any other month?