AST 120 Activity 18
Earth's Orbit and the Second Law of Planetary Motion

Name	Full	Partial	None

We have seen that Kepler used an eccentric/equant construction to describe the orbits of the planets. Once he got access to Tycho's Mars data he was able to select a series of four measurements of Mars' location to determine the parameters of its orbit. This was an incredibly difficult process, and Kepler filled 900 folio pages with calculations written in small handwriting to determine an orbit that would match the four observations. Finally, he found an orbit that fit the observed positions to within 2', better than the accuracy of the observations themselves. Success! But not so fast ...

Kepler wanted to verify his orbit by comparing his calculated positions to several other observations that Tycho made. The news was not good. The calculations differed from the observed positions by as much as 8'. Kepler knew that Tycho's observations were good to 4', so an 8' error was simply unacceptable.[1]

Kepler decided to start over. He wanted to get let the observed positions of Mars tell him what the correct orbit was, without assuming anything in advance (like that the orbit was a circle, etc.). But to do this he first needed a better theory for the orbit of Earth. Let's see how he tackled this problem...

1. The technique that Kepler used to map out Earth's orbit was basically the same technique that is used in surveying land, and it is known as triangulation. The problem is, for triangulation to work you need *two* fixed landmarks. As a Copernican, Kepler could count on the Sun to serve as one of these landmarks. But what could serve as the other landmark? In what Einstein would later cal "an idea of true genius" Kepler realized that he could use *Mars* as his second landmark. How? By using the locations of Mars at times spaced 687 days apart. Kepler knew that Mars had an orbital period of 687 days, so after this period of time it would return to its starting point and this could serve as a "fixed" landmark! Now let's see how he used this idea to determine Earth's orbit. Kepler extracted from Tycho's data observations of the position of Mars and the Sun taken on the same day. He found three sets of observations that were spaced 687 days apart. The table below shows the a set of data similar to what Kepler used.

Day	Longitude of Mars	Longitude of Sun
0	♉ 2°	♉ 28°
687	♈ 4°	♐ 2°
1374	♉ 23°	♌ 7°

2. On the zodiacal chart on the last page of this handout, draw a line representing the line of sight from Earth to Mars on Day 0. Think about how you need to draw this line and check with Dr. T after you have drawn it.

[1]Such a small error might have been perfectly acceptable to someone constructing a mathematical fiction that could be used to predict the locations of the planets. But it was not acceptable to Kepler who wanted to know the *true* motions of the planets so that he could understand the divine Creator.

3. Now draw the line of sight from Earth to the Sun on Day 0. Draw a noticeable dot at the point where this line intersects the first line.
4. What *must* be at the intersection of these two lines?
5. Repeat this process for Day 687.
6. Repeat the process for Day 1374.
7. Now you have found three locations of Earth on its orbit. Kepler used three such locations to construct a circular orbit for Earth. To do this he needed to determine a circle that would pass through all three points. The key is to find the center of the circle. One way to do this is to pick a pair of points and construct the *perpendicular bisector* of the line segment that joins those points. Do this for one pair of points. If you aren't sure how to construct the perpendicular bisector using a compass and straightedge, just ask Dr. T.
8. Now choose another pair of points and construct a second perpendicular bisector.
9. The intersection of the two perpendicular bisectors is the center of the circle. Use the compass to accurately draw the Earth's circular orbit.
10. Once you have drawn the orbit, look at the angle (as measured from the center of the orbit) between the location of Earth on Day 0 and the location on Day 687. Now look at the angle between the location on Day 687 and the location on Day 1374. Are these two angles equal? If not, which is greater?
11. According to this data, does Earth move along this circular orbit at a uniform rate? Explain your answer.
12. We have already seen that Kepler initially used an equant construction, but ultimately become convinced that the correct description of the motion was that the speed of the planet varies inversely with its distance from the Sun. He needed a way to *calculate* where the Earth would be on its orbit at any given time. While trying to work this out he noticed something interesting. We can see what he noticed by running the program `SecondLawCircle`. The simulation shows a planet (Earth, let's say) orbiting the Sun on an eccentric circle with speed inversely proportional to the distance from the Sun. The green dot shows the planet's location now, while the blue dot shows its location a short time (let's say six days) in the past. The red pie-slice (or sector) connects these two dots with the Sun. Plots show the area of the sector and the distance of the planet from the Sun as a function of time. How much does the distance change? Record the maximum and minimum distances below, as well as their percent difference ((max-min)/max).

max distance = ______________________________

min distance = ______________________________

% difference = ______________________________

13. Does the area of the sector change? Record the maximum and minimum areas, as well as their precent difference.

max area = ______________________________

min area = ______________________________

% difference = ______________________________

14. These results suggest that the area changes much less than the distance does. Note that when the planet is far from the Sun the sector is long and skinny, but when it is close to the Sun the sector is short and fat, thus maintaining a roughly constant area. [2] Why is the sector skinny when the planet is far from the Sun?

15. Why is the sector fat when the planet is close to the Sun?

16. This simulation greatly exaggerates the eccentricity of Kepler's orbit for Earth. What happens if we use a more realistic eccentricity? Pause the simulation, Reset the View, change the eccentricity to 0.04 (don't forget to hit Return) and then hit Play. How much does the distance change? Record the maximum and minimum distances below, as well as their percent difference ((max-min)/max).

max distance = ______________________________

min distance = ______________________________

% difference = ______________________________

17. Does the area of the sector change? Record the maximum and minimum areas, as well as their precent difference.

max area = ______________________________

min area = ______________________________

% difference = ______________________________

18. Is the area of the sector really constant? Is it approximately constant?

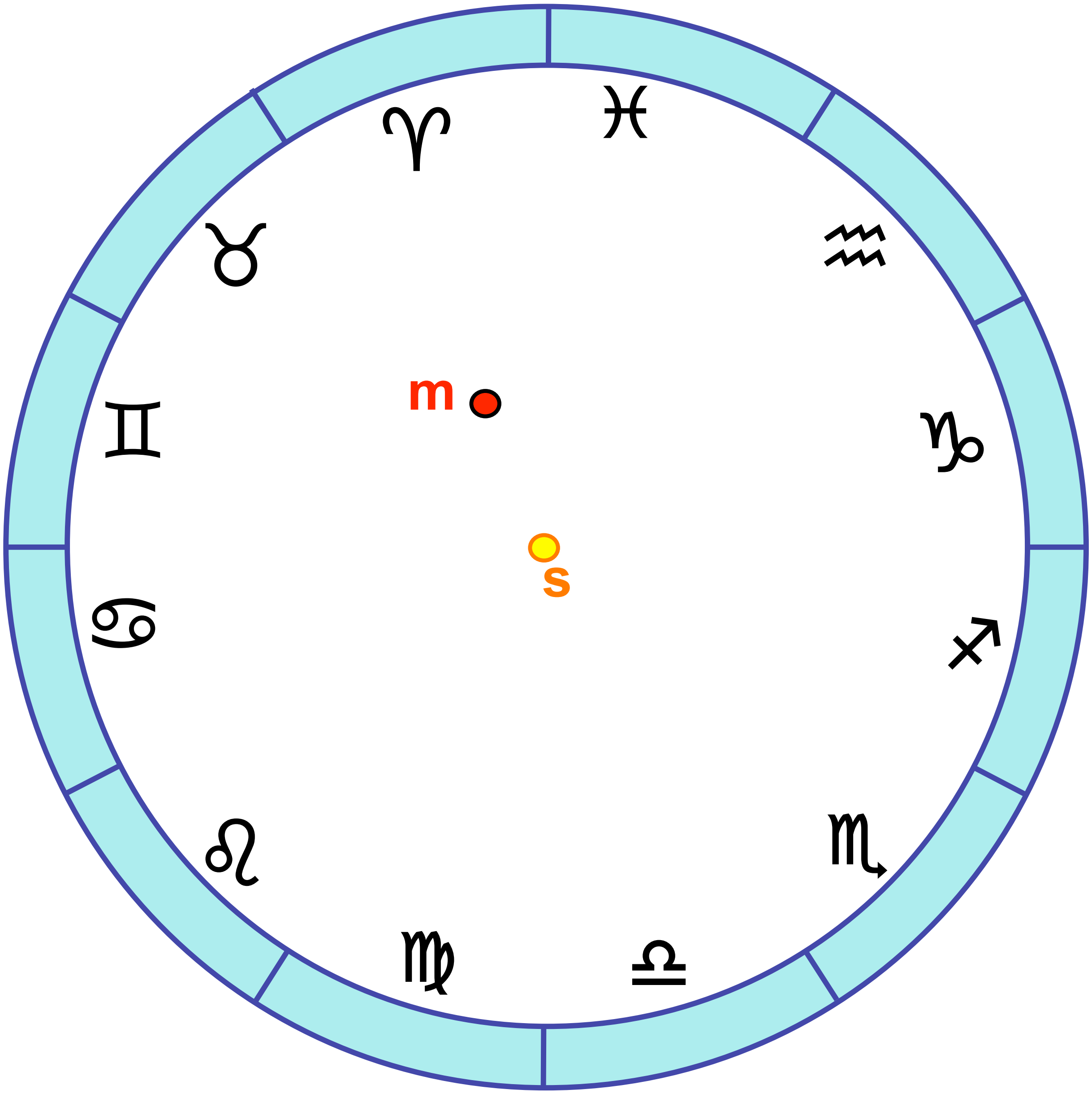
[2] Kepler thought he had proved that the area was exactly constant for such an orbit. In fact, we now know it *is* constant for the correct orbit, but the correct orbit is not a circle and the speed isn't inversely proportional to distance from the sun. More on this later.

19. Kepler believed he had proved that the line connecting a planet to the Sun would sweep out equal areas in equal intervals of time. This statement has come to be known as "Kepler's Second Law of Planetary Motion" (it's his second law because it appears in his book after the First Law, but he actually found the Second Law first). We now believe that the Second Law holds true for a single planet orbiting the Sun - but we also know that Kepler's "proof" of the Second Law is incorrect. Still, it was a critical step forward in the development of celestial physics. In any case, it gave Kepler a means of computing the position of the Earth in its orbit at any time. To see how this works, suppose the radius of Earth's orbital circle (ie the distance from the center of the circle to the Earth) is 1 AU. How much area (in AU^2) will the line connecting the Earth and Sun sweep out over the course of a full orbit (365.25 days)?

20. How much area will the line sweep out over the course of 25 days?

21. So if we know the location of the Earth at some time we can find where the Earth will be 25 days later by drawing the line connecting the Earth and Sun and letting it sweep out the correct amount of area. When we reach the right area the line will be pointing from the Sun to the location of the Earth after 25 days. In space below, sketch a diagram of what the "swept out" area might look like for a span of 25 days.

22. Now go back and add to your diagram another sector that shows the area swept out over a span of 25 days, but in a different part of the orbit. How do the areas of the two sectors compare?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