# Kepler's Laws

Laboratory 8

Astronomy 120. The Copernican Revolution

| **Name** | **Full** | **Partial** | **None** |
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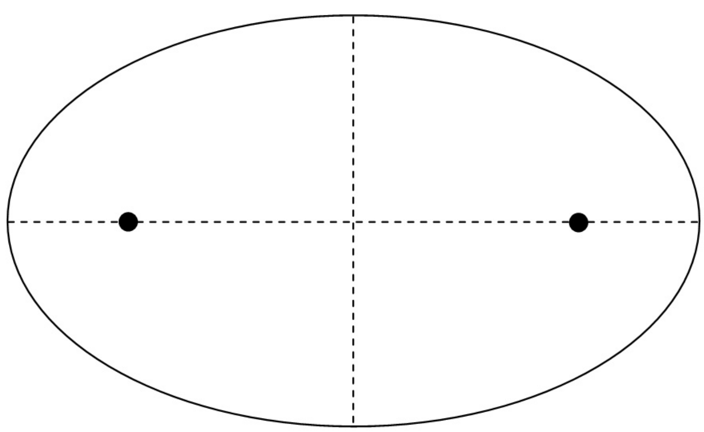
## Summary

Reviews Kepler's three laws, determine the properties of an ellipse and relate these properties to the orbits of planets. Determine how various properties of a planet's orbit affect its observed motion through the sky. Explore how Kepler's three laws, combined with some observational data, serve to explain the motions of all planets in exquisite detail.


## Review of Kepler's Laws

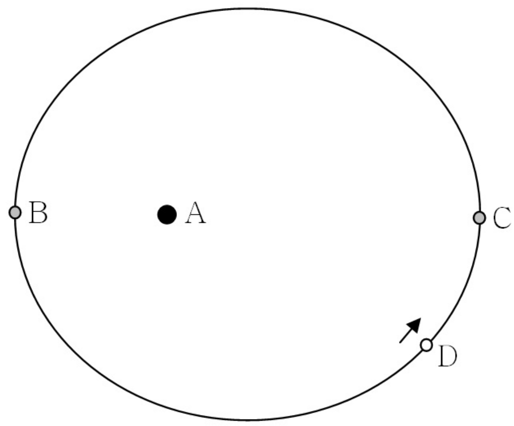
1. Refer to the textbook and clearly label the parts (focus, semi-major axis, semi-minor axis, center) on the following diagram of an ellipse:



2. The *eccentricity* of an ellipse is defined as the distance from a focus to the center of the ellipse divided by the length of the semi-major axis. Calculate the eccentricity of the ellipse in the figure above: ____________________________

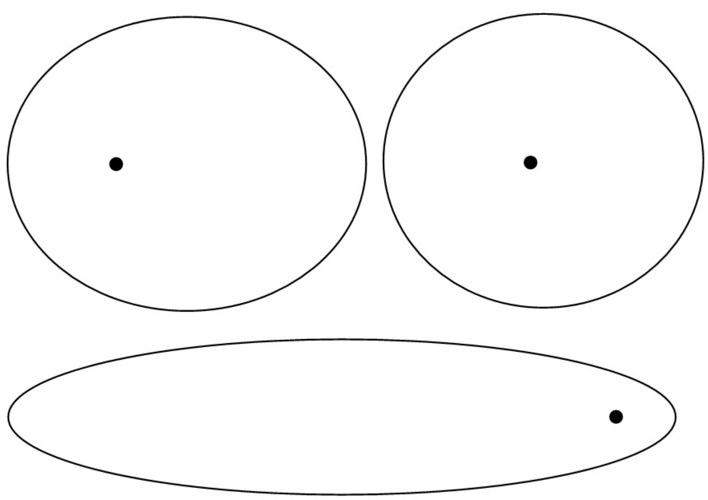
3. State Kepler's First Law in your own words:

4. Examine the following figure of a planet orbiting the Sun and fill in the blanks with the correct letter:




- Focus ______________
- Aphelion ______________
- Perihelion ______________
- Planet has greatest speed ______________
- Planet has lowest speed ______________
- Planet's speed is increasing as it moves from ______________ to ______________
- Planet's speed is decreasing as it moves from ______________ to ______________

5. Calculate the approximate eccentricities for the following ellipses:



top left $e =$ ______________

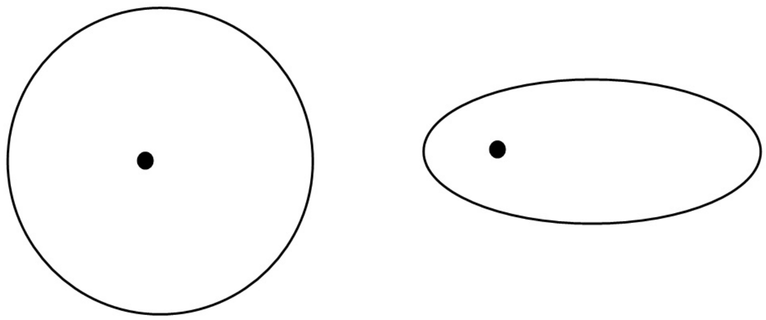
top right $e =$ ______________

bottom $e =$ ______________

6. State Kepler's Second Law in your own words:

7. Kepler's Third Law relates the ____________________________ ($T$) it takes a planet to go around the Sun in an orbit of a given ____________________________ ($a$). The relationship between $T$ and $a$ is: ____________________________

8. The following two orbits have the same value for $a$. How do their values of $T$ compare? Explain your answer.



9. Use Kepler's second law to determine for which of the two orbits shown above (left or right) the planet will take the greatest amount of time to move from aphelion to each of the following points. Circle the correct response (where "same" means that both planets reach the point at the same time).

   (a) first quadrant: right   left   same

   (b) perihelion: right   left   same

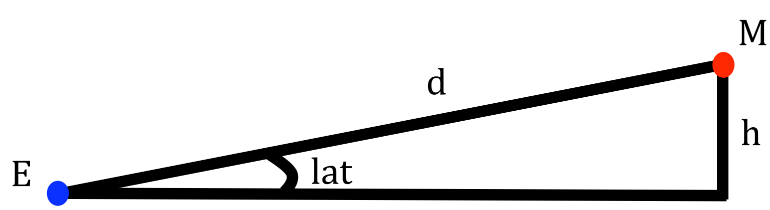
   (c) third quadrant: right   left   same

10. True story: Galileo saw the planet Neptune in the background during one of his observations of Jupiter, and he actually noted that it seemed to have moved a bit over several days' time. However, Jupiter quickly passed it and he did not return to Neptune. If he had stuck with it, he may have discovered the 8th planet before the 7th (Uranus) was found, and after watching it awhile he might have estimated its period to be about 160 years. If he had had Kepler's third law in hand, he could have also estimated its average distance from the Sun; what average distance would he have found? Show your work.

### Planetary Latitudes

Now let's look at how all this gets put together to explain the observed motion of the planets. Run the `KeplerSystem` program. Hit the play button to play the simulation. The top window shows the orbit of Earth and one other planet (initially set to match the orbit of Mars) around the Sun. The orbits are ellipses (Kepler's First Law) and each planet moves on its orbit in such a way that the line from the Sun to the planet sweeps out equal areas in equal times (Kepler's Second Law). In addition, the periods and semimajor axes of the Earth and the other planet are related via Kepler's Third Law ($T^2 \propto a^3$). One other important feature to note is that while Earth orbits in the ecliptic plane (indicated by the grid), the other planet (Mars) orbits in a plane that is tilted with respect to the ecliptic. Use the mouse to change the view of the simulation until you can clearly see that Mars' orbit is tilted with respect to the ecliptic plane. Kepler had a specific reason for giving Mars this tilt: it would explain the variation in Mars' ecliptic *latitude*. [1] So far we have only been paying attention to the ecliptic longitude of the planets. We just assumed that the planets all move in the ecliptic plane. But our observations from Starry Night show us that this isn't so. Ptolemy and Copernicus both knew this, and they came up with systems for predicting the latitude of a planet as well as its longitude. But their latitude systems were *significantly different from their longitude systems*. To explain changes in latitude both Ptolemy and Copernicus introduced additional circles that were not part of their theory for longitudes. Kepler was trying to find the *actual orbit* of the planet through space. So for him a single orbit should explain variations in both longitude and latitude. Kepler found that he could accomplish this goal by simply tilting his elliptical orbits relative to the ecliptic plane. This strategy worked perfectly!

1. Let's think a bit about what determines a planet's ecliptic latitude (as seen from Earth), and how this all fits together in Kepler's system. The diagram below shows the Earth (`E`) and Mars (`M`). The side labeled `h` shows the distance of Mars above the ecliptic plane, while the hypotenuse of the triangle (labeled `d`) shows the distance from Earth to Mars. The other side of the triangle would then represent the distance from Earth to Mars measured *along the ecliptic plane*.




In the space below write a trigonometric equation that gives the correct relationship between `h`, `d`, and the latitude angle `lat`.

[1] Recall that ecliptic longitude measures a planet's apparent location along the ecliptic, while ecliptic latitude measures how far above or below the ecliptic the planet appears to be.

2. The ecliptic latitude of the planet will be greatest when _____________.
   (a) `h` is small and `d` is large
   (b) `h` is small and `d` is small
   (c) `h` is large and `d` is small
   (d) `h` is large and `d` is large
3. This means that in general we would expect to see larger latitudes for Mars when Mars is in _____________.
   (a) western quadrature
   (b) eastern quadrature
   (c) opposition
   (d) conjunction
4. Think back to what you know about how the planet's appear to move through the sky. How will Mars appear to be moving when it has its maximum latitude?
5. Now let's assume that Mars is doing whatever you said it should be doing in the last two questions in order for it to have maximum ecliptic latitude. Will it *always* have a large latitude when it is doing these things? If not, what other thing determines how big Mars' latitude will be?
6. Now let's go back to the computer simulation. Clear the traces and watch the apparent motion of Mars in the bottom window. We measure ecliptic longitude from west to east, which will be from right to left in the Ecliptic View Frame. The vernal equinox is at the center of the window, and the autumnal equinox is at the right (and left) side. Let's measure the ecliptic longitude from the `VE`, so call the right side of the window $-180°$ and the left side $180°$ longitude. Watch the simulation for a while to see when Mars has its maximum possible (positive) latitude. What is Mars' ecliptic longitude when its latitude is at the maximum?
7. How does Mars appear to move when its ecliptic latitude is at the maximum?
8. Is Mars bright or dim when its ecliptic latitude is at the maximum?

9. Continue watching the simulation to determine when Mars' latitude is at its minimum (largest negative value). What is the longitude of Mars when its latitude is at the minimum?

10. How is Mars moving when its latitude is at the minimum?

11. Is Mars bright or dim when its ecliptic latitude is at the minimum?

12. Based on your observations, explain how Mars' orbit is tilted relative to the ecliptic. In what direction is it tilted upward? In what direction is it tilted downward?

13. Explain how you could determine both the direction and the angle of the tilt of Mars' orbit from observations of the apparent position of Mars in the sky.

Kepler's ability to predict the longitudes and latitudes of the planets using a single orbit is an example of a *unification* in science. A unification occurs when two theories, developed for explaining two different sets of phenomena, are shown to really be two aspects of a single *unifying* theory. Unification is perhaps one of the most stunning examples of *progress* in science. Both realists and phenomenalists see unification as progress. Realists, like Kepler, see unifications as evidence that we are getting at the truth of things. Phenomenalists, like Osiander, wouldn't see it that way. But they would still be impressed by the great simplification that is brought about by unifying two theories into one.

Exploring the Keplerian System

Kepler's theory of planetary motion is sometimes considered to be "Copernican." Indeed, Kepler thought of himself as Copernican (and the textbook he wrote to explain his theories is titled *Epitome of Copernican Astrnomy*). But Kepler's theory is radically different from the system of Copernicus. He has done away with the uniform circular motion that Copernicus held so dear and replaced it with ellipses and the Law of Areas. He has discarded the geometric constructions of Copernicus in favor of physical explanations. He uses a single orbit to predict longitudes and latitudes where Copernicus constructed two separate sets of circles. In fact, the "Keplerian" system has only two things in common with the Copernican system: in both systems the Sun doesn't move, and both systems use the same ordering of the planets.

In fact, Kepler's new astronomy was initially not well received, even by Copernicans. Galileo, who would become the most prominent and outspoken Copernican scientist of his era (other than Kepler), never believed in Kepler's elliptical orbits. But what eventually won the day for Kepler was the incredible accuracy of the predictions based on his theory. The three figures below show errors in predictions of the longitude of Mercury using three different systems. The top figure shows errors of predictions based on the Ptolemaic *Alfonsine Tables*, the middle figure shows errors from the Copernican *Prutenic Tables*, and the bottom figure shows errors from the Keplerian *Rudolphine Tables.*

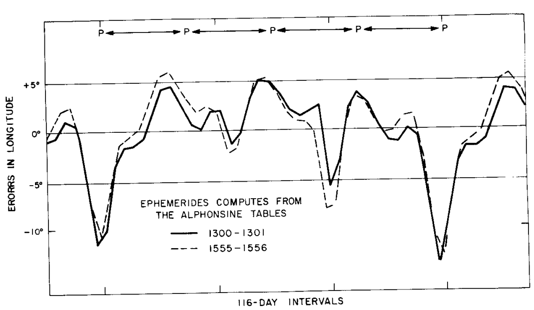



**Figure 3.** *Errors in the longitude of Mercury as predicted by the Ptolemaic* Alfonsine Tables *during the Middle Ages.*

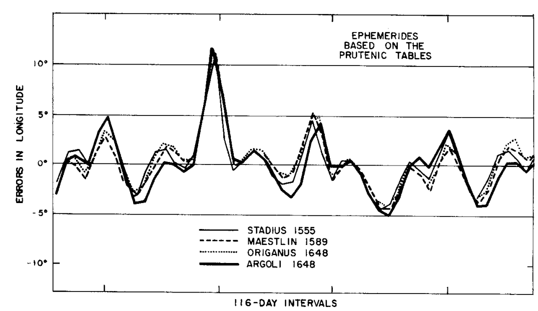



**Figure 5.** *Errors in the longitude of Mercury as predicted by four astronomers who based their ephemerides on the Copernican* Prutenic Tables.

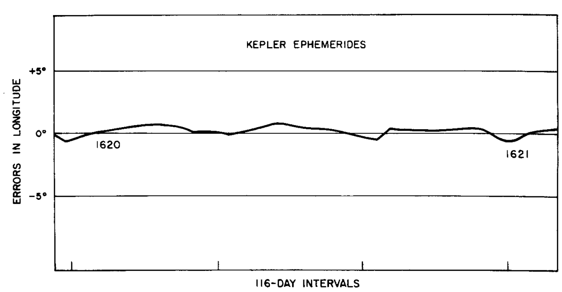



**Figure 6.** *Errors in the longitude of Mercury were greatly reduced by Kepler, who based the predictions on his* Rudolphine Tables.

1. Based on these figures explain why professional astronomers would likely adopt the Keplerian system for predicting the ecliptic longitude of a planet.

2. The table below gives the parameter values that are used to simulate the orbits of all five planets in the `KeplerSystem` program. You don't have to type these in, just select the planet you want from the Select Planet menu. Take a look at the motion for each planet and then answer the remaining questions.

| **Planet** | | Semimajor Axis | Eccentricity | Apsidal Angle | Earth Apsidal Angle | Inclination |
|---|---|---|---|---|---|---|
| Mercury | ☿ | 0.387 AU | 0.206 | 299.3° | 324.6° | 7.01° |
| Venus | ♀ | 0.718 AU | 0.007 | 324.9° | 296.2° | 3.39° |
| Earth | ♁ | 1.00 AU | 0.017 | NA | NA | 0° |
| Mars | ♂ | 1.524 AU | 0.093 | 196.5° | 323.3° | 1.85° |
| Jupiter | ♃ | 5.20 AU | 0.049 | 185.1° | 272.4° | 1.31° |
| Saturn | ♄ | 9.58 AU | 0.056 | 246.0° | 259.3° | 2.49° |

3. Which planet has the greatest eccentricity?

4. Which planet has the least eccentricity?

5. Which planet has the greatest inclination? Does this planet also appear to have the largest deviations from the ecliptic?

6. Which planet has the least inclination? Does this planet always stay very close to the ecliptic?

7. Explain why Mars shows larger deviations from the ecliptic than Saturn, even though Saturn has a greater inclination than Mars.