AST 120 Activity 13
The Scale of the Universe

Name	Full	Partial	None

In the last activity we saw how Copernicus determined the period of each planet's revolution around the Sun. In this activity we will examine how Copernicus determined the ordering of the planets and the sizes of their orbits. For this purpose we will continue to use the `CopernicanSystem` program, so you should go ahead and run that program now.

1. We have seen that Copernicus' system provides a clear distinction between inferior and superior planets. In the space below, briefly explain why, in the Copernican system, a planet whose orbit is smaller than that of Earth can *never* be seen in opposition to the sun and *must* be brightest and retrograde when it is in conjunction.

2. Which planets have orbits smaller than that of Earth?

3. In the Select Planet menu, choose User Defined. Try different values for the radius of the planet's orbit (but all less than 1, since we want to look at inferior planets). [Note: since we aren't changing the angular speed along with the size, you may get odd behavior like no retrograde motion - but don't worry about that right now.] Pay close attention to the maximum elongation of the planet from the sun in the Sky View window. How is the maximum elongation related to the size of the planet's orbit?

4. What is the greatest possible maximum elongation for any conceivable inferior planet? How big must the orbit of the planet be in order to most closely approach this greatest possible value?

5. We know that Venus has a maximum elongation of about 48°, while Mercury has a maximum elongation of about 28°. Which of these two planets has the larger orbit?

6. In fact, with a bit of trigonometry we can determine the size of each inferior planet's orbit compared to the size of Earth's orbit. Select Venus from the Select Planet menu, then play the simulation until the planet reaches maximum (eastern or western) elongation. You should find that the lines connecting the Earth, the planet, and the Sun form a right triangle. Sketch the arrangement below (you don't need to draw the orbit circles, just the triangle). Label the line connecting the Sun and the Earth with the symbol R_E. Label the line connecting the Sun and Venus with the symbol R_V. Indicate that the angle at Earth's location is 48° (since this is the maximum elongation of Venus as seen from Earth - make sure you understand why it is *this* angle that is 48°).

7. Write down an equation (using a trigonometric function) that relates the 48° angle to the lengths R_E and R_V. You may want to consult Appendix B of the textbook.

8. Let's let $R_E = 1$ AU (the AU, or Astronomical Unit, is defined to be equal to the mean Earth-Sun distance). Solve the equation you wrote down above to determine R_V. Record your answer (with units) below.

9. Now determine the radius of Mercury's orbit (with units) and record your result below.

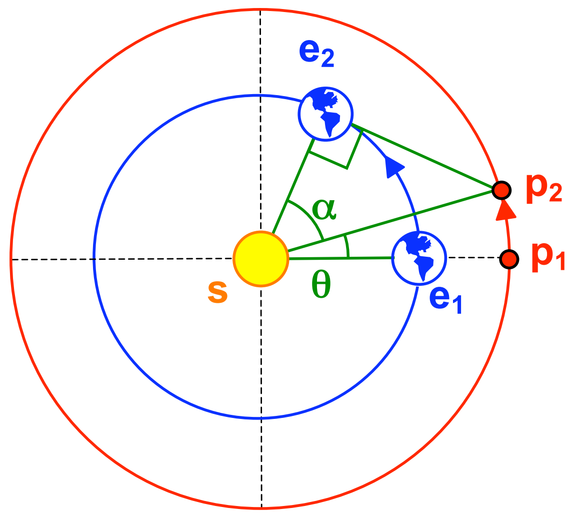
10. Now let's look at superior planets. In the space below, briefly explain why, in the Copernican system, a planet whose orbit is larger than that of Earth can be seen in opposition to the sun and *must* be brightest and retrograde at that time.

11. Based on what you know of the actual apparent motions of the Sun and five planets, which planets *must* be superior planets in the Copernican system?

12. Determining the relative distances of the superior planets is more challenging. Select Mars from the Select Planet menu. Play the simulation and pause it when the planet reaches opposition. Sketch the arrangement of Mars, Earth, and Sun in the space below.

13. Now play the simulation until Mars reaches quadrature (eastern or western). Recall that quadrature means that planet appears to be 90° away from the Sun in the sky as seen from Earth. So the angle at the location of Earth should be 90°. Sketch this arrangment below.

14. If we put the two pictures together we should get something like this ...



In the figure above, label the distance from the Earth to the Sun R_E and the distance from Mars to the Sun R_M. In the space below, write an equation (using a trigonometric function) that relates R_E and R_M to the angle α.

15. Now it turns out that we can't measure α directly form observations. But we can still figure it out. First we can find the angle θ. By observation we can determine that it takes 106 days for Mars to move from opposition to quadrature. Through what fraction of a full orbit has Mars moved in this time? Recall that the orbital period for Mars (the time it takes for Mars to go all the way around its orbit) is 686 days.

16. Since 360° is a complete orbit, how many degrees has Mars moved through in 106 days? In other words, what is the angle θ in degrees?

17. Now note that the angle $\theta + \alpha$ is just the angle through which Earth has moved in this same 106 days. Determine the value of $\theta + \alpha$ in degrees and record the result below.

18. Now we can find α by subtracting the θ from $\alpha + \theta$. Determine alpha and then use your equation from above to find R_M. Let $R_E = 1$ AU. Show your work below.

19. Now you can follow the same procedure to determine the distances from the Sun to Jupiter and Saturn. First determine the angles θ, $\theta + \alpha$, and α (defined as above) for these two planets. The time between opposition and quadrature (t_Q) and the orbital period (T) is given for each planet in the table below. Complete the table using the procedure you used above for Mars.

Planet	t_Q	T	θ	$\theta + \alpha$	α
Jupiter	87.5 days	4283 days			
Saturn	86.9 days	10613 days			

20. Use the same equation you used for Mars and the appropriate value of α from the table above to find the distance from the Sun to Jupiter (R_J). Let $R_E = 1$ AU. Show your work below.

21. Now find the distance from the Sun to Saturn (R_S). Let $R_E = 1$ AU. Show your work below.

22. Let's put all of your results together. Record the orbital periods you determined in the last activity and the orbital radii you found in this activity in the table below. Record the orbital periods in (Earth) years (ie divide the number of days by 365). Record the orbital radii in AU.

Planet	Orbital Period (years)	Orbital Radius (AU)	Orbital Speed (AU/yr)
Mercury ☿			
Venus ♀			
Earth ♁			
Mars ♂			
Jupiter ♃			
Saturn ♄			

23. So far things look pretty good. One nice feature of the Copernican system is that the farther a planet is from the Sun the longer its orbital period is. This makes sense because the farther away the planet is the farther it has to travel to go around its orbit. But there is even more to it. Let's calculate the orbital *speed* for each planet. To do this you first need to find the circumference of the planet's orbit. Recall that the circumference is $2\pi R$, where R is the radius. Calculate the circumference and then divide by the period to find the speed in AU/yr. Record your results for each planet in the table above.

24. Do all planets move at the same speed? If not, is there some other pattern you notice? Describe any relation you find between a planet's distance from the sun and its orbital speed.

25. So now we know, according to Copernicus, how far away all of the planets are. But recall that Copernicus had to assume that the stars were very far away. Download and run the `EarthOrbit` program again. In the Display Options menu select Trace Motion of Celestial Poles. Adjust the radius of Earth's orbit until you think the circles traced out by the celestial poles are small enough that we would not notice them. Record the value for the radius of Earth's orbit (given in the simulation) below.

26. In the `EarthOrbit` simulation, the celestial sphere has a radius of 1 unit. Compare this to the radius of Earth's orbit you found in the previous question. If the orbit of Earth is small enough that we don't notice the movement of the celestial poles (over the course of a year) then what must be the minimum radius of the Celestial Sphere in AU?

27. Compare the minimum radius for the Celestial Sphere to the Copernican radii for the planets as recorded in the table above. How does the radius of the celestial sphere compare to these other radii? Why do you think astronomers in the 16th century might have found this objectionable?

28. We have now completed our main investigation of the Copernican system. Now, for a moment, try to think like a 16th century astronomer or natural philosopher. What do you think are the major advantages of the Copernican system, as compared to the Ptolemaic system?

29. Still thinking like a 16th century astronomer or natural philosopher, what do you think are the major disadvantages of the Copernican system, as compared to the Ptolemaic system?