Ptolemy's Universe

Laboratory 5

Astronomy 120. The Copernican Revolution

Name	Full	Partial	None

Introduction

Here is what we know so far about Ptolemy's theories for the motion of the planets:

1. The deferent/epicycle model can produce retrograde motion and it automatically (naturally) accounts for the connection between retrograde and brightness.

2. Ptolemy could ensure that superior planets retrograde at opposition by synchronizing the motion of the planet along the epicycle with that of the Sun.

3. Ptolemy could ensure that inferior planets stay close to the Sun by synchronizing the motion of the epicycle center along the deferent with that of the Sun (thus keeping the center of the epicycle on the Earth-Sun line).

4. The period of the motion along the deferent is just the planet's zodiacal period.

5. The period of the motion along the epicycle (measured relative to the deferent) is just the planet's synodic period.

6. Only the *ratio* of the size of the epicycle to the size of the deferent matters. If we rescale both circles by the same factor there will be no observable change.

7. Ptolemy employed the eccentric and equant devices to account for changes in the apparent speed of the planets across the sky.

In this lab we are going to focus on a simplified version of the Ptolemaic models, ignoring the eccentric and equant. Our goal is to determine the relative sizes of the epicycle and deferent for each planet. In other words, we want to find out the value of R_e/R_d for each planet, where R_e is the radius of that planet's epicycle and R_d is the radius of that planet's deferent. Once we have determined the R_e/R_d ratios, we can add one more assumption used by Ptolemy and build up a picture of the entire Universe as envisioned by Ptolemy and his followers.

1. Let's start by trying to find R_e/R_d for an inferior planet. Run the `Inferior Ptolemaic` simulation. Play the simulation for Venus. Do your best to pause the simulation when Venus is at its maximum elongation from the Sun. Sketch the arrangement of the circles below. Make sure to indicate the Earth, the deferent, the epicycle, Venus, the Sun's orbit, and the Sun in your sketch. Label the angle that corresponds to Venus' maximum elongation with a θ_{max}.

2. In your diagram above, draw line segments from Earth to the center of the epicycle, the center of the epicycle to Venus, and Venus to Earth. These line segments form a triangle. Redraw the triangle in the space below. Label the line segment from Earth to the center of the epicycle with R_d (because it is a radius for the deferent circle). Label the line segment from the center of the epicycle to Venus with R_e (because it is a radius for the epicycle). Label the angle θ_{max} as above. One of the other angles in this triangle is a right angle. Indicate which angle is a right angle in your diagram.

3. Use trigonometry (see App. B) to write an equation that relates the angle θ_{max} to the ratio R_e/R_d.

4. We know that Venus has a maximum elongation of 48°. Use this information to determine the ratio R_e/R_d for Venus.

5. The geometry works the same way for Mercury. The only difference is that Mercury has a maximum elongation of only 28°. Determine the ratio R_e/R_d for Mercury.

6. Now we have found the ratios R_e/R_d for both inferior planets. Let's turn our attention to the superior planets. This is a bit harder. Quit `Inferior Ptolemaic` and run the `Superior Ptolemaic` program. Play the simulation (if necessary) for Mars until Mars is exactly in opposition to the Sun. Draw a diagram showing this situation in the space below. Make sure to indicate the Earth, the deferent, the epicycle, Mars, the Sun's orbit, and the Sun in your sketch.

7. Now play the simulation again and pause it when Mars is in quadrature. Recall that if Mars is in quadrature then that means Mars will appear to be 90°away from the Sun as seen from Earth. Sketch the situation in the space below. Make sure to indicate the Earth, the deferent, the epicycle, Mars, the Sun's orbit, and the Sun in your sketch. Draw the line segment from the Earth to the Sun, from the Earth to the center of the epicycle, from the center of the epicycle to Mars, and from Mars to Earth. TWO of the angles in your diagram are right angles. Indicate which angles are right angles in your diagram. Check with your instructor to make sure your diagram is correct before moving on.

8. In your diagram for the previous question three of the line segments form a right triangle. Copy this right triangle into the space below. Also include the line segment from the Earth to the Sun. Label the vertex of the triangle that correspond's to Earth's location `e`, the vertex that corresponds to Mars `m`, and the vertex that corresponds to the center of the epicycle `c`. Label the Sun's location `s`. Label the angle $\angle\texttt{cem}$ with θ. Label the line segment $\bar{\texttt{ec}}$ with R_d (because $\bar{\texttt{ec}}$ is a radius for Mars' deferent). Label the line segment $\bar{\texttt{cm}}$ with R_e (because $\bar{\texttt{cm}}$ is a radius for Mars' epicycle).

9. Use trigonometry (see App. B) to write an equation that relates the angle θ to the ratio R_e/R_d.

10. OK, this is great, we found an equation for R_e/R_d just as we did for the inferior planets. But what is θ? Unfortunately, θ is not an angle we can directly measure. But we CAN figure it out, because we can determine the angles through which the Sun and Mars have moved between our two pictures above. Look back at your two diagrams. When Mars was in opposition the angle `sec` was 180°. When Mars is in quadrature the angle `sec` is $90° + \theta$. The *difference* between these two angles must be due to a difference between how much the point `s` has moved and how much the point `c` has moved, which is an angle we will call α. So we find that:

$$180° - (90° + \theta) = \alpha.$$

Solve this equation for θ in terms of α and write your result in the space below.

11. Suppose the *time* between these two pictures (ie, the time from when Mars is in opposition to when it is in quadrature) is t_Q. Let the period of the Sun's motion around Earth (which is one year, of course) be denoted T. During the time T the Sun moves 360°on its orbit. Through how many degrees does the Sun move in a time t_Q? Give an expression for this angle in the space below. Ask your instructor for help if you need it.

12. We also need to determine the angle through which the center of the epicycle has moved during this time. The center of the epicycle moves around the deferent with a period equal to the planet's zodiacal period, which we will denote by T_z. So during a time T_z the epicycle center moves 360°. Through how many degrees does it move in a time t_Q? Give an expression for this angle in the space below.

13. Now we want to find the difference between the angle through which the Sun has moved and the angle through which the epicycle center has moved. That difference is the angle we have called α. Use your results from the last two questions to write an expression for α in the space below.

14. For Mars, the time from opposition to quadrature is $t_Q = 106$ days. The zodiacal period of Mars is $T_z = 686$ days. Of course, the period of the Sun's orbit is $T = 365$ days. Find the value of α for Mars.

15. Use this value of α to find the value of θ for Mars.

16. Now use your value for θ and the equation you derived above to find R_e/R_d for Mars.

17. OK, so that was a lot harder than it was for inferior planets. But now that you have walked through it step by step, let's see if you can apply the same procedure to find R_e/R_d for Jupiter and Saturn. Complete the table below.

Planet	t_Q	T_z	$360°t_Q/T$	$360°t_Q/T_z$	α	θ	R_e/R_d
Jupiter	87.5 days	4283 days					
Saturn	86.9 days	10613 days					

18. That was a lot of work, so you might want to rest for a minute. Once you are ready to go again, we will try to figure out how to put all of these deferents and epicycles together to build up a picture of the entire Universe. To do this we need to include an important assumption that Ptolemy made in his book *The Planetary Hypotheses*. In that book he assumed that the deferents and epicycles were really equators of solid spheres that were rotating around in the heavens. He wanted to make sure that the spheres for one planet did not overlap with those for another, but otherwise he wanted everything as tightly packed as could be so as to avoid having any empty space (recall that Aristotle said a vacuum could not exist). So Ptolemy assumed that the maximum distance from Earth to a given planet must be equal to the minimum distance from Earth to the next planet out.

    To understand how to use this assumption we must first be able to determine the *ratio* of the maximum distance to the minimum distance for a single planet. The table below will help you do this. For each planet we assume that $R_d = 1$. Then we use our values for R_e/R_d to find the value of R_e. The minimum distance from Earth to the planet is just $d_{min} = R_d - R_e$ (assuming all deferents are centered on Earth). The maximum distance is just $d_{max} = R_d + R_e$. Then the ratio of the maximum distance to the minimum distance is just d_{max}/d_{min}. Complete the table below to determine these ratios. [Note that I have included the Sun and Moon, which in our simple model do not have epicycles.]

Planet	R_d	R_e	d_{min}	d_{max}	d_{max}/d_{min}
Moon	1	0	1	1	1
Mercury	1				
Venus	1				
Sun	1	0	1	1	1
Mars	1				
Jupiter	1				
Saturn	1				

19. Using a method known as *trigonometric parallax* (which we will discuss in detail later in the course) the Ancient Greeks had determined that the distance from the Earth to the Moon was about 30 times Earth's diameter (or $30D_E$). Since there is no empty space between the sphere of the Moon and the sphere of Mercury, the minimum distance to Mercury should be equal to the distance to the Moon. We can then find the maximum distance to Mercury using the ratio we found in the table above. This maximum distance to Mercury will be equal to the minimum distance to Venus, and so on. Complete the table below to determine the minimum and maximum distance to each planet.

Planet	**Minimum Distance** (in D_E)	**Maximum Distance** (in D_E)
Moon	30	30
Mercury	30	
Venus		
Sun		
Mars		
Jupiter		
Saturn		

20. Ptolemy thought that the maximum distance to Saturn would be equal to the distance to the Celestial Sphere. How many times greater is the *diameter* of the Celestial Sphere than the diameter of Earth?

21. Use this information, and the fact that the volume of a sphere is directly proportional to the cube of its diameter, to find how many times greater the volume of Celestial Sphere is than the volume of the Earth (according to Ptolemy).