# AST 120 Activity 20
# Kepler's *Harmonices Mundi*

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| Name | Full | Partial | None |
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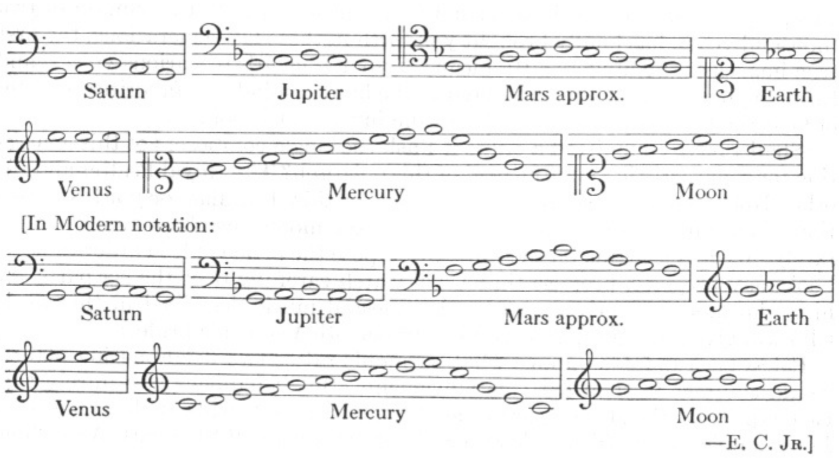
In one of his final works Kepler returns to the ideas expressed in his *Mysterium Cosmographicum.* By this time, years after the publication of the *Astronomia Nova*, Kepler has completely revamped planetary astronomy with his elliptical orbits and his Law of Areas. He has made accurate determinations of the orbits of all six planets (which he presented in his textbook *Epitome of Copernican Astronomy*). He is now armed with better information than he had when writing the *Mysterium.* Unfortunately, he finds that his new distances don't agree with the Platonic solid ratios much better than the old distances did. He doesn't give up on this idea, but takes it to be only an approximate rule. The Creator was guided by the principle of the regular solids, but not ONLY by this principle. Kepler's ultimate goal in his final book,[1] Kepler explains that the REAL principle underlying the Creation is musical harmony.

He explains the idea of harmony as it relates to music, geometry, astrology, and finally astronomy. His main astronomical idea is that the speed of a planet's motion along its orbit is related to the pitch of a musical note. The range from the planet's maximum speed (when it is at perihelion) to its minimum speed (when it is at aphelion) produces a range of musical notes. Together the planets produce celestial music. The figure below shows the sets of musical notes that Kepler uses to represent the motion of each planet, so each planet has its own little motif. It's not exactly Mozart, but still Kepler thought the planetary motions represented the *mathematical* principles of harmony at their best (he goes through many pages of argument to explain why these harmonies are the best ones) .




[1]After *Harmonices Mundi* Kepler published his astronomical tables (the "Rudolphine Tables") and what is essentially the first science fiction novel, known as the *Somnium.* He also published a second edition of the *Mysterium.* But the *Harmoncies Mundi* represents his last new astronomical treatise.

One interesting note (pardon the pun) is that Kepler's main resource for his musical research was a treatise written by the Italian musician Vincenzo Galilei. Vincenzo's son, Galileo, will enter our picture shortly.

The most historically important result in the *Harmonices Mundi* is what has now come to be known as Kepler's Third Law of Planetary Motion. It is delivered as an off-hand remark: "But it is absolutely certain and exact that *the ratio which exists between the periodic times of any two planets is precisely the ratio of the 3/2th power of the mean distances.*" We might express this idea as an equation:

$$T \propto a^{3/2}$$

where $T$ is the period of a planet's orbit and $a$ is the mean distance of the planet from the Sun (actually we should use what is called the semi-major axis of the planet's elliptical orbit). If we square both sides of the equation we find:

$$T^2 \propto a^3.$$

This is but one example of a "harmony of the spheres" out of many examples that Kepler gives. But this one is of great historical importance since it played a key role in the development of Newton's theory of gravity.

Kepler didn't provide any evidence for this assertion in *Harmonices Mundi*, but it's pretty easy to verify. The table below shows the periods of the planets (essentially as determined by Copernicus) and the lengths of the semi-major axes of their elliptical orbits (as determined by Kepler from Tycho's data). Units for period are years, and units for lengths are AU. That way both $T$ and $a$ have the value 1 for Earth, which makes the constant in our proportionality relationship equal 1 as well. Compute $T^2$ and $a^3$ for each planet and record your results in the table.

| **Planet** | | $T$ **(years)** | $a$ **(AU)** | $T^2$ **(yrs$^2$)** | $a^3$ **(AU$^3$)** |
|---|---|---|---|---|---|
| Mercury | ☿ | 0.241 | 0.388 | | |
| Venus | ♀ | 0.616 | 0.724 | | |
| Earth | ♁ | 1.00 | 1.00 | | |
| Mars | ♂ | 1.88 | 1.524 | | |
| Jupiter | ♃ | 11.86 | 5.20 | | |
| Saturn | ♄ | 29.46 | 9.51 | | |

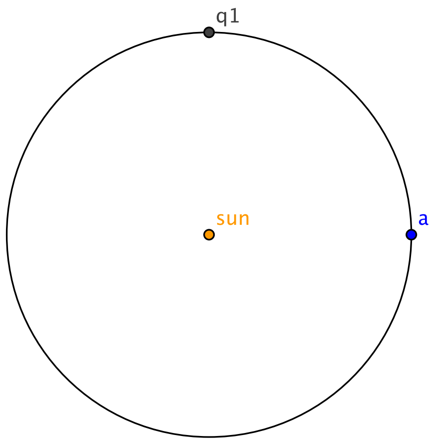
1. Based on this data, would you say that Kepler's Third Law holds?

2. Halley's comet orbits the sun just like the planets do. Assuming the comet follows the same orbital laws as the planets, what is the shape of its orbit?

3. The comet has an orbital period of 76 years. Use Kepler's third law to find the length of the semimajor axis of the comet's orbit (in AU).

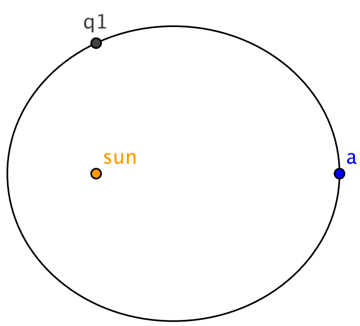
4. Suppose there was another planet between Mercury and Venus. If the orbit of this planet has a semimajor axis of 0.55 AU, then what must be the period of this planet's orbit (in years)? What is the period in days?

5. Suppose the planet has a nearly circular orbit like the one shown below. Shade the region swept out by the planet-sun line as the planet moves from aphelion (`a`) to first quadrant (`q1`). Approximately what fraction of the orbits total area is swept out?




6. How many days does it take for this planet to travel from aphelion to first quadrant, assuming it has the nearly circular orbit shown in the previous question?

7. Now suppose the planet has an elongated orbit like the one shown below. Shade the region swept out by the planet-sun line as the planet moves from aphelion (`a`) to first quadrant (`q1`). Approximately what fraction of the orbits total area is swept out?




8. How many days does it take for this planet to travel from aphelion to first quadrant, assuming it has the nearly circular orbit shown in the previous question?

9. A planet with a perfectly circular orbit (with the sun at the center) takes ______________ of its orbital period to travel from aphelion to first quadrant.

   (a) more than 1/4

   (b) exactly 1/4

   (c) less than 1/4

10. A planet with an elliptical orbit (with the sun at one focus) takes ______________ of its orbital period to travel from aphelion to first quadrant.

    (a) more than 1/4

    (b) exactly 1/4

    (c) less than 1/4

11. Comets (like Halley's comet) have very elongated orbits. Most comet orbits have semi-major axes that are greater than that of Saturn's orbit. Draw the orbit of a comet and mark the position of the Sun. Mark the perihelion, the aphelion, and the octants of the comets orbit (first, third, fifth, and seventh). Shade the area swept out by the Sun-comet line between the third and fifth octants (which includes perihelion). Do the same for the area swept out between the seventh and first octant (which included aphelion). Does the comet spend more time near perihelion (between 3rd and 5th octants) or near aphelion (between 7th and 1st octant)? Explain your answer using the Law of Areas.