AST 120 Activity 16
Johannes Kepler's *Mysterium Cosmographicum*

Name	Full	Partial	None

Johannes Kepler began working as a mathematics teacher at a seminary in Graz (in modern Germany) in 1594. While teaching he had an amazing idea about the relationships between planetary orbits in the Copernican system. He wrote a treatise explaining this idea in 1595, and in 1596 it was published under the title *Mysterium Cosmographicum.*[1] Although Kepler's big idea turned out to be a dead end, it was incredibly influential on his later work which would ultimately usher in a new era in astronomy. We will explore Kepler's beautiful, but ultimately unworkable, idea in this Activity.

1. While lecturing at the blackboard Kepler drew a circle circumscribing an equilateral triangle (a "regular" three-sided polygon - regular means all 3 sides and all 3 angles are the same), in order to illustrate the 60° separation between the locations of successive conjunctions of Jupiter and Saturn. While drawing this figure it occurred to him that if another circle was drawn inscribed within the triangle that the ratio of the radii of these two circles would be fixed. If you make the triangle bigger, both circles get bigger by the same factor (so the ratio of their radii doesn't change). Kepler wondered if something like this might explain the relative sizes of the orbits of the planets (he was a Copernican, and could thus treat the orbits as being almost circles). To get an idea of what Kepler was up to, run the `MysteriumCosmographicum2D` program. Select Triangle from the pull-down menu. This shows an equilateral triangle and two circles (red and blue). The radii of the circles can be adjusted using the sliders at the bottom. Adjust the radii until one circle is inscribed within the triangle (this circle just barely fits inside the triangle) and the other circle circumscribes the triangle (the triangle just barely fits inside the circle). Record the two radii below, and calculate their ratio (larger divided by smaller).

   Smaller radius = ____________________

   Larger radius = ____________________

   Ratio = ____________________

2. Now select Square from the pull-down menu. This shows a square (the regular polygon with 4 sides) and two circles with adjustable radii. Adjust the radii so that one circle is inscribed within the square while the other circumscribes the square. Record the radii and their ratio below.

   Smaller radius = ____________________

   Larger radius = ____________________

   Ratio = ____________________

[1] Actually it had a much longer title, but this is what everyone calls it.

3. Now select Pentagon from the pull-down menu. This shows a pentagon (the regular polygon with 5 sides) and two circles with adjustable radii. Adjust the radii so that one circle is inscribed within the pentagon while the other circumscribes the pentagon. Record the radii and their ratio below.

   Smaller radius = ____________________

   Larger radius = ____________________

   Ratio = ____________________

4. To see if these ratios match the ratios of planetary orbits we need to know the ratios of the orbits in the Copernican system. The table below shows these ratios:

Outer Planet	**Inner Planet**	**Ratio of Orbital Radii**
Saturn	Jupiter	1.57
Jupiter	Mars	3.00
Mars	Earth	1.32
Earth	Venus	1.26
Venus	Mercury	1.38

   Do the ratios from the triangle, square, and pentagon fit with the ratios in this table?

5. Kepler realized quickly that there was a problem with his idea. The first few ratios didn't match well, and he realized that there would be an infinite number of ratios to try because there are an infinite number of regular polygons. He dropped his initial idea because he came up with an even better one. The world is three-dimensional, not two-dimensional. So Kepler thought that it made more sense to consider *spheres* circumscribed around (or inscribed within) regular *polyhedra*. Regular polyhedra are often called *Platonic solids*. They are solid figures constructed of identical regular polygons. The sides of the polygons are joined to form edges of the solid. Where the edges come together they form a vertex. Each vertex must be formed by the same number of edges for the solid to be considered regular. A simple example is a cube. The faces of the cube are squares (a regular polygon). Each vertex (corner) of the cube joins three edges together. Now try to construct a Platonic solid of your own. Dr. T will give you instructions for how to construct your Platonic solid. Describe the solid your group built below. How many faces does it have? What shape are the faces? How many of these shapes come together at each vertex?

6. Let's compile the information on the solids that were constructed by all the groups in the table below.

Name	Faces	Shape of Face	Faces at Each Vertex

7. Are there any other regular solids that we could create? Could we put six triangles together at a vertex? What happens? What happens with seven triangles at a vertex? How about four squares? Four pentagons? Could any of these work to produce a new regular solid?

8. Could we put three hexagons (six-sided regular polygons) together at a vertex to construct a regular solid? Note that the interior angle at each corner of a hexagon is $120°$.

9. So we see that there are five, and only five, regular solids. This is what got Kepler so excited. In the Copernican system there are six planets (counting the Earth). So there are five "spaces" between planets. Kepler imagined that one of the regular solids would fit perfectly between each pair of neighboring planetary spheres (so that one sphere circumscribes the solid while the smaller sphere is inscribed within the solid). If so, this would *explain* why there were six, and only six, planets. But to make this idea work he had to show that the ratios of the radii for the circumscribing and inscribed spheres would match the calculated ratios of the orbital radii for the planets. To explore this idea run `MysteriumCosmographicum3D`. Select Tetrahedron from the pull-down menu. Adjust the radii of the two spheres until one sphere is inscribed within the tetrahedron and the other sphere circumscribes the tetrahedron. Note: you will need to view the simulation from a variety of angles to get this right. Record the radii and their ratio below. There a particular way to orient the tetrahedron in order to find the radius for the inscribed sphere. See if you can figure out the trick and describe it below.

10. Now record your radii for the inscribed and circumscribed spheres and calculate their ratio (larger divided by smaller).

    Smaller radius = ____________________

    Larger radius = ____________________

    Ratio = ____________________

11. Select Cube from the pull-down menu. Repeat the procedure to measure the circumscribing and inscribed radii. Record your values and their ratio below.

    Smaller radius = ____________________

    Larger radius = ____________________

    Ratio = ____________________

12. Select Octahedron from the menu. Repeat the procedure to measure the circumscribing and inscribed radii. Record your values and their ratio below.

    Smaller radius = ____________________

    Larger radius = ____________________

    Ratio = ____________________

13. Select Dodecahedron from the menu. Repeat the procedure to measure the circumscribing and inscribed radii. Record your values and their ratio below.

Smaller radius = ______________________________

Larger radius = ______________________________

Ratio = ______________________________

14. Select Icosahedron from the menu. Repeat the procedure to measure the circumscribing and inscribed radii. Record your values and their ratio below.

Smaller radius = ______________________________

Larger radius = ______________________________

Ratio = ______________________________

15. Let's try to match these ratios up with the ratios of orbital radii given above. Fill in the table by matching the ratio for each pair of planets with the closest ratio for one of the Platonic solids.

Outer Planet	**Inner Planet**	**Ratio of Orbital Radii**	**Platonic Solid**	**Ratio of Spheres**
Saturn	Jupiter	1.57		
Jupiter	Mars	3.00		
Mars	Earth	1.32		
Earth	Venus	1.26		
Venus	Mercury	1.38		

16. You may have had a hard time putting the Octahedron and Cube anywhere. Their ratios don't seem to match up with anything, except maybe the Saturn-Jupiter ratio. Kepler had a way around this. Instead of finding the ratio of the circumscribing sphere to the inscribed sphere for the octahedron, he replaced the inscribed sphere with a sphere that was inscribed in the central square of the octahedron. This sphere actually pokes out of the octahedron itself, but it fits neatly within the square the forms the octahedron's middle. Return to the `MysteriumCosmographicum3D` program to find the radius of this sphere, then recompute the ratio.

Smaller radius = ______________________________

Larger radius = ______________________________

Ratio = ______________________________

17. Now, using this new ratio for the Octahedron instead of the one you computed originally, fill in the table again. Match the ratio for the planets to the nearest ratio for a Platonic solid.

Outer Planet	**Inner Planet**	**Ratio of Orbital Radii**	**Platonic Solid**	**Ratio of Spheres**
Saturn	Jupiter	1.57		
Jupiter	Mars	3.00		
Mars	Earth	1.32		
Earth	Venus	1.26		
Venus	Mercury	1.38		

18. What do you think? Does this seem to work? Was Kepler onto something here?