Eratosthenes measures the Earth

Laboratory 3

Astronomy 120. The Copernican Revolution

Introduction

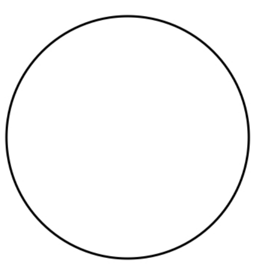
In trying to understand the cosmos, the ancient Greeks used simple geometry to dazzling effect. Around 200 B.C., the astronomer Eratosthenes conceived a way to measure the size of the Earth by simple observation. A brilliant polymath, Eratosthenes had left his native Cyrene in present-day Libya to study in Athens before moving to the intellectual hotbed of the Mediterranean: Alexandria, Egypt. He was appointed tutor to King Ptolemy III's son, Philopater, and eventually became the director of the famous Alexandrian Library. (King Ptolemy was not related to Claudius Ptolemy, the author of the *Almagest*.)

Eratosthenes learned of a curious phenomenon that occurred once a year in the Egyptian village of Syene (pronounced si-e′-ne, now the city of Aswan), just below the Nile's first cataract, some 5,000 stadia (roughly 500 miles) south of Alexandria: At noon on the first day of summer, the Sun stood directly overhead in Syene; a vertical stick cast no shadow and the Suns rays penetrated all the way to the bottom of a deep well on nearby Elephantine island, illuminating the water below. Eratosthenes was intrigued by the report because at the identical hour in Alexandria, the noontime Sun did not stand at the zenith of the sky, but approximately 7 degrees away. The Sun's oblique rays created a shadow beside a vertical stick and struck the wall, not the bottom, of the local wells. From this information, Eratosthenes calculated the size of the Earth, just as you are going to do now. Show all your work and your answers on the attached worksheet.

Dividing the circle

First, some practice with circles and angles. Fact: A circle contains 360 degrees. Dividing a circle will yield pie-shaped sectors whose degree measures must add up to 360 degrees.

1. On the diagram below, draw a line dividing the illustrated circle in half. What do you think is the degree measure of each sector? Now divide the circle so it has four equal sectors. What do you think is the degree measure of each of these sectors? Divide the circle further into eight equal sectors. What do you think is the degree measure of each of these "one-eighth" sectors? Record the degree measures in the table next to the diagram.



No. of sectors	degree measure of each sector
2	
4	
8	

2. Based on your results from part (1), write down a general rule on how to compute the degree measure of each sector if given the number of sectors in a circle.

3. Write down the "reverse" rule, that is, the rule on how to compute the number of sectors in a circle if given the degree measure of each sector.

4. Using your rule from part (3), compute the number of sectors in a circle whose sectors are each 7 degrees wide. "Round off" your answer to the nearest tenth. Draw a circle around your answer; you'll need it later.

Parallel Lines

When he was figuring out the size of the Earth, Eratosthenes had to make an assumption: that the Earth is so small and so far away from the Sun that the Sun's rays are nearly parallel when they hit the Earth. That is, the Sun's rays illuminate Alexandria and Syene from essentially the same direction. We can demonstrate this by examining a scale model of the Earth and Sun. We now believe that the Sun's diameter is about 100 times that of the Earth. We have also determined that the Earth is roughly 100 Sun diameters from the Sun.

5. Run the AngularSize program in the Copernican Revolution package. This program shows the Earth and the Sun, with light rays emanating from the center of the Sun to the top and bottom points on the Earth. We want to examine how the angle between these two light rays changes as we move the Earth farther from the Sun and shrink the Earth in comparison to the Sun. First use the Distance from Sun slider to increase the distance of Earth from the Sun. Increase the distance until it is equal to 100 solar diameters. What happens to the angle between the two light rays as the Earth gets farther away?
   (a) It gets considerably smaller.
   (b) It gets a little bit smaller, but doesn't change much.
   (c) It does not change at all.
   (d) It gets a little bit larger, but doesn't change much.

7. Now reduce the Diameter of Earth (using the other slider) until it is 100 times smaller than that of the Sun (i.e. until it is 0.01 solar diameters). What happens to the angle between the two light rays when you shrink the Earth?
   (a) It gets considerably smaller.
   (b) It gets a little bit smaller, but doesn't change much.
   (c) It does not change at all.
   (d) It gets a little bit larger, but doesn't change much.

8. With the simulation set to the proper scale (distance is 100 solar diameters, diameter of Earth is 0.01 solar diameters) what is the angle between the two light rays shown in the simulation? Record your answer below (in degrees). Then convert your answer to minutes of arc. Finally, convert your answer to seconds of arc.

9. If instead of looking at these two light rays you picked two light rays hitting at other points on the Earth would the angle between those rays be larger or smaller?

10. Is such a small angle even measurable using primitive tools (like a gnomon and its shadow)? Was it reasonable for Eratosthenes to assume that the light rays hitting Earth from the Sun were all parallel?

11. Eratosthenes had heard that on the first day of summer, at local noon, the Sun was directly overhead in Syene. Like good scientists, we would like to verify this claim before accepting it as the truth. Start up Stellarium. Get into the Locations window (compass rose icon on left) and find Syene. The modern name for Syene is Aswân, Egypt. Just type "Asw" and you should see this city on the list that appears. Select this city and then get out of the Locations window. Turn on the meridian (;), turn off the atmosphere (a) and fog (f). Use the Date/Time window to set the date to June 21, 250 BC (when Eratosthenes was a young man). We want to set the time to noon, but to do this we need to account for the fact that clocks in Egypt are 7 hours ahead of ours. So set the time to 5AM. The first thing we must do is determine the time of local noon at Syene. Find the sun and track (t) it. Adjust the time in the Date/Time window by minutes until the Sun is centered on the Meridian in the alt-az grid. Record the time (which will be the time displayed plus 7 hours) in the space below.

12. What is the Sun's altitude from Syene (Aswân) at local noon on June 21, 250 BC? To get an accurate determination of the altitude from Stellarium just click on the sun and read the Alt value shown. Record the Sun's altitude to the nearest minute of arc in the space below.

13. So is it true that the Sun is directly overhead from Syene at local noon on the summer solstice?[1]

14. Now go back to the Locations window and find Alexandria. The modern Arabic name for this city is al-Iskandarîyah. Just type in "al-Isk" and you should see it in the menu. Select the city and then get out of the Locations window. Find and track (t) the sun. Determine the time for local noon on June 21, 250 BC in Alexandria. Record this time below (remember to account for the 7 hour time difference).

[1]Eratosthenes failed to account for two things. One is that the report of the Sun being directly overhead at Syene dated from a long time before Eratosthenes' day, and a change in the obliquity (tilt) of the ecliptic had displaced the Tropic of Cancer a bit to the South in the intervening time. The second thing Eratosthenes failed to account for is that the Sun is not a point but a disk with finite size. So there are actually a range of locations for which the Sun is "directly overhead" at any given time.

15. Is Alexandria due North of Syene? If not, which city is farther west? How can you tell?[2]

16. Determine the altitude of the Sun from Alexandria at local noon on June 21, 250 BC. Record your result to the nearest minute of arc below.

17. What is the difference in the Sun's altitude at local noon between Alexandria and Syene?

18. Recall from our gnomons lab that the shadow cast by a gnomon can be used to determine the altitude of the Sun in the sky. We saw in that lab how the shadow changes *as the Sun moves in the sky*. Now we want to see how the shadow changes *as we move the gnomon on the Earth's surface*. Quit Stellarium. Run the Eratosthenes program from the Copernican Revolution package. This simulation shows two gnomons and the rays from the Sun that strike the top of the gnomon (and thus form the top of the shadow). For one of the gnomons the Sun is located directly overhead (as Eratosthenes thought was true for Syene on the summer solstice). The other gnomon can be moved around on the Earth's surface using the Angle slider. Increase the Angle slider and watch what happens to the shadow cast by the second gnomon. What happens to the shadow?

    (a) It gets larger.
    (b) It remains the same size.
    (c) It gets smaller.
    (d) There is no shadow.

[2]This is another problem for Eratosthenes measurement, because he assumed that the distance between Alexandria and Syene was measured in a purely North-South direction. Luckily for him this error tends to cancel with the ones mentioned earlier, leading to an accurate result in spite of these mistakes. We will, like Eratosthenes, ignore all of this and pretend that the Sun is directly overhead at Syene on the summer solstice and that Syene is due South of Alexandria.

19. Eratosthenes knew that the Sun's rays made an angle of 7° with a gnomon at local noon on the summer solstice in Alexandria. He believed that at the same time in Syene a gnomon would cast no shadow at all. Based on these two pieces of data he could reasonably conclude that ____________.

    (a) the gnomon in Alexandria is tilted 7° relative to the gnomon in Syene

    (b) the gnomon in Alexandria is a little longer than the gnomon in Syene

    (c) the gnomon in Alexandria is closer to the Sun than the gnomon in Syene

    (d) the gnomon in Alexandria is tilted 83° relative to the gnomon in Syene

20. In the Display Option menu select Show Angle Between Gnomons. Set the Angle in the simulation to about 30° and leave it there (if we set it to 7° it is too hard to see everything). In the space below draw a large circle to represent the Earth. Draw the setup in the simulation on this diagram (i.e. show the two gnomons, the two rays of sunlight, and the shadow of the second gnomon). Extend the lines for the gnomon to the center of the Earth. Extend the lines for the sun's rays across the entire diagram (remember that these rays should be parallel). (Note: the lines are easier to see if you deselect Show Earth in Display Options.) Label the angle made by the two gnomon lines at the center of the Earth with the symbol θ. Label the angle between the second gnomon and the sun ray passing through its top with the symbol ϕ.

21. How are θ and ϕ related? Explain your answer.

22. Eratosthenes determined from observation that the angle ϕ was about 7°. So what is the angle θ?

FINISHING UP

Now you can figure out the size of the Earth as Eratosthenes did.

23. Note that the angle θ defines a 7-degree-wide sector of the Earth's entire circle. Referring to your results from part (1), how many such sectors make up the entire circle of the Earth?

24. Each 7-degree sector corresponds to a 500-mile arc on the Earth's surface, that is, the separation between Alexandria and Syene. Therefore, to compute the Earth's circumference, all you need to do is multiply 500 miles by the number of 7-degree sectors you determined in part (1). How many miles are in the circumference of the Earth?

    Circumference of the Earth = ________________________

25. Therefore, what is the Earth's diameter? (Recall that the circumference of a circle is π times the circle's diameter.)

    Diameter of the Earth = ________________________

26. If you have time, reopen Stellarium and show that the angle between Syene and Alexandria could have been found by measuring the altitude of Polaris from both locations *on any night.* Record the altitude of Polaris from Syene today, as well as the altitude of Polaris from Alexandria today. Show that their difference is about 7°.

27. **Challenge**: We saw earlier that Eratosthenes overestimated the (angular) difference in latitude between Syene and Alexandria because he thought Syene was on the Tropic of Cancer (where the sun passes directly overhead on the summer solstice) when in fact it is about 0.5° north of the Tropic line. You can check this yourself. Look up the latitude for the Tropic of Cancer and the latitude for Aswân (Syene) and you will find that they differ by about half a degree. We also saw that Syene is not due South of Alexandria, even though Eratosthenes assumed that it was. Explain why these two errors tend to cancel each other out rather than add together.