AST 120 Activity 22
Galileo on Falling Bodies

Name	Full	Partial	None

By this point in our story the *astronomical* evidence for the Copernican (or, more correctly, Keplerian) viewpoint has become overwhelming. But there is still the problem of the physics. Aristotle's physics clearly indicates that the Earth cannot move, and if it was somehow moving we would see all sorts of things that we just don't see. To finally overcome 1400 years of tradition we must find a new physics, one that will apply to both the sublunary and celestial spheres. The great pioneer in this endeavor was Galileo, and we will begin our discussion of Galileo's new physics with the topic of bodies falling from rest.

1 Galileo's Predecessors

1. Recall that Aristotle's law of motion is

$$v = \begin{cases} kF/R, & F > R \\ 0, & F \leq R. \end{cases}$$

   where v is the speed of the object's motion, F is the force on the object, R is the resistance of the medium, and k is just a proportionality constant. In the case of an object falling from rest the force is just the object's weight and the resistance is due to the air. So according to Aristotle a 3-lb ball should fall ______________ a 1-lb ball.

   (a) three times faster than

   (b) one third as fast as

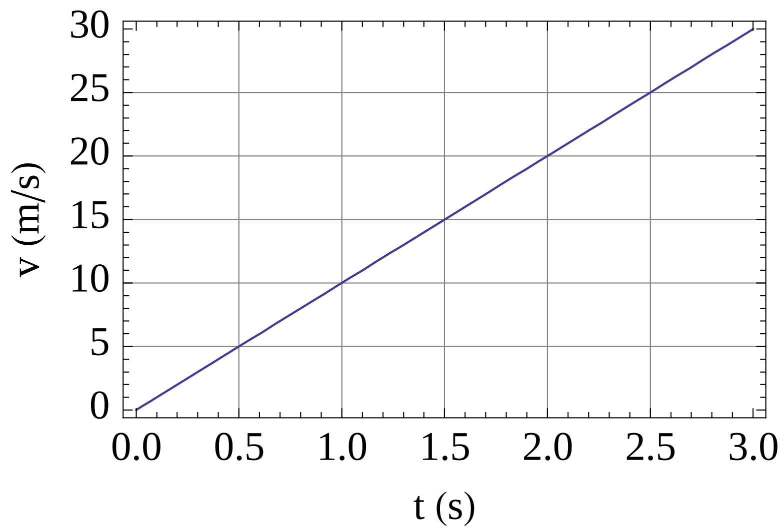
   (c) at the same speed as

2. When Aristotle talks about the speed of an object, he really means what we would now call *average speed*. For a falling object the average speed would be the distance the object falls divided by the time it takes the object to fall. So if a 3-lb ball and a 1-lb ball are dropped from the same height and the 3-lb ball hits the ground after 6 seconds, the 1-lb ball will hit the ground after ______________ seconds.

3. Aristotle's law of motion implies that an object will fall at a constant speed (since the force, and presumably the resistance, are constant). So the fall of an object should be *uniform*. A precise definition of uniform motion was given by John of Holland in 1369: uniform motion is motion in which "a body traverses an equal space in every equal part of time." So if objects fall in uniform motion then an object that falls 10 feet in 5 seconds would fall ______________ feet in 3 seconds.

4. Aristotle actually gives some indication in his writings that objects might increase their speeds as they fall. By the 14^{th} Century it was widely recognized that objects speed up continually as they fall. Albert of Saxony, a scholar at Oxford University, suggested two ways this could occur: the speed could increase proportional to the distance the object has fallen ($v \propto d$) or the speed could increase proportional to the time the object has been falling ($v \propto t$).[1] Note that we have introduced a new conception of speed here: *instantaneous speed*. Instantaneous speed refers to the speed of an object at a particular point in space, or a particular instant of time, rather than an average speed over some interval. For example, suppose an object starts from rest and after one second its speed is 5 m/s. If its speed is proportional to time then after 3 seconds its speed is ______________ m/s.

5. If an object is moving uniformly (at constant speed) we know how to determine the distance it has traveled after any period of time. But what if it is moving so that its speed increases proportional to the time? In 1377 Nicole Oresme, of the University of Paris, found a way to determine the distance in this case as well. He came up with the idea of representing the speed at different times using a graph. An example of the type of graph he used is shown below.



The speed of this object at $t = 1$ s is ______________.

6. Is the object represented by this graph moving such that $v \propto t$? Explain how you can tell.

7. Oresme realized that the distance traveled by an objects is given by the *area under the curve* in the speed versus time graph. This is fairly obvious for the case of uniform motion (for which the line in the graph would be horizontal and the area under it would be simply vt). But he held that it would also work for nonuniform motions. Determine the area under the curve in the graph above. Note: the area of a triangle is one half the base times the height. Make sure to account for units.

[1] Albert ultimately rejected both of these possibilities because they could lead to infinite speeds.

8. Using this idea Oresme proved the *mean speed theorem*, which states that an object moving such that $v \propto t$ will travel a distance equal to the average of its initial and final speeds times the time of travel. Determine the mean (or average) of the initial and final speeds for the object depicted in the graph above.

9. Now use the mean speed theorem to find the distance traveled by the object depicted in the graph. How does your answer compare with the area under the graph you found earlier?

10. Draw a dashed horizontal line in the graph above showing the value of the mean speed. In the space below, explain how you can see from the graph that the area under this horizontal line is equal to the area under the solid line. This is basically how Oresme proved the mean speed theorem.

11. Also in the 14^{th} Century, William Heytesbury (another Oxford scholar - most of the important new ideas on motion in the 14th Century came from two places: Oxford and Paris) provided a definition of the concept of acceleration. Basically he stated that acceleration is the rate at which an object's speed changes. With this definition, we can see that motion with $v \propto t$ is really just motion with *constant acceleration*, and the proportionality constant is a measure of the acceleration. So we can rewrite the proportionality as an equation: $v = at$, where a is the acceleration of the object. If an object falls from rest with constant acceleration of 3 m/s^2, its speed after 5 seconds of falling will be ______________.

12. Suppose an object starts from rest, so $v_i = 0$. It experiences a constant acceleration a for a time t. What is the object's final speed v_f? Note: give a symbolic answer in terms of a and t.

13. Now let's try putting together Heytesbury's idea of acceleration with Oresme's mean speed theorem. Note that the mean speed theorem can be written as: $d = (v_i + v_f)t/2$. Make sure you see how this equation relates to the statement of the mean speed theorem given above. Substitute your expression for v_f from the previous question into the mean speed theorem formula to find the distance traveled by the object (in terms of a and t).[2]

[2] This result too was known in the XIVth Century. It appears in a manuscript attributed to William Collingham of Oxford.

14. So an object moving with a constant acceleration travels a distance which is proportional to the ______________.
    (a) time
    (b) square of the time
    (c) square root of the time
    (d) time cubed

2 Galileo's Theory of Falling

Most of the key ideas for describing the motion of falling bodies were formulated in the 14^{th} Century, but the medieval Scholastics do not seem to have conducted experiments to see if these ideas actually fit any motions that exist in nature. That work would await Galileo in the 17^{th} Century. It is uncertain how much Galileo was aware of the work of the 14^{th} Century scholars at Oxford and Paris. In any case, he came to many of the same conclusions they did and went well beyond their work. His ideas about motion were laid out in a final form in his *Discourses and Demonstrations Concerning Two New Sciences* in 1638.

1. Galileo begins his discussion of falling bodies by attacking Aristotle's views. He starts with a logical argument. Suppose you drop an 1-lb stone and find it takes 10 seconds to fall to the ground. According to Aristotle it should take the 10-lb stone only 1 second to fall to the ground. Now suppose we tie these two stones together and drop them. It seems as though the slowly falling 1-lb stone will impede the fall of the 10-lb stone. So the time of fall should be ______________.
    (a) less than 1 s
    (b) exactly 1 s
    (c) between 1 s and 10 s
    (d) exactly 10 s
    (e) more than 10 s
2. Now suppose instead we fuse the two stones together into a single 11-lb stone. How long will it take this stone to fall to the ground, according to Aristotle's physics?
    (a) less than 1 s
    (b) exactly 1 s
    (c) between 1 s and 10 s
    (d) exactly 10 s
    (e) more than 10 s
3. There seems to be a logical flaw in Aristotle's theory. Furthermore, Galileo argues that Aristotle's theory completely fails the test of experiment. Let's test Aristotle's idea ourselves. On your table you should find a ball made of wood and a ball made of lead. Pick up both balls and get a sense of their weights. Which one is heavier? Which one will fall faster? According to Aristotle, approximately how much faster will the heavier ball fall (just estimate, you don't have to determine this exactly)?

4. Now drop both balls at the same time from the same height. Be careful to drop the balls simultaneously and from exactly the same height above ground. Have another member of your group closely watch the spot where the balls will land. Which ball lands first? Or do they land at the same time? How does this result compare with Aristotle's theory?

5. Galileo claimed that experiments dropping lead balls of different weights from a tower show that the heavier weight hits the ground when the lighter weight is a "hands breadth" above the ground. So there is a difference in the time of fall, but not so great a difference as Aristotle claimed. Galileo claimed that this difference was due to the resistance of the air. Do your experimental results fit with Galileo's?

6. Having destroyed Aristotle's theory of falling, Galileo begins searching for a new theory to replace it. He first considers the various ways that an object might move. It was known since the 14th Century that falling objects don't have uniform motion, so Galileo next considers uniform acceleration. He develops the theory of uniformly accelerated motion that we have already discussed above (specifically $v = at$ and $d = (1/2)at^2$). Galileo didn't use algebraic formulas like these. Instead, he thought in terms of ratios. If an object starts from rest and moves with uniform acceleration, then if the object has traveled a distance d_1 at time t_1 and a distance d_2 at time t_2 these values must be related by
   (a) $(d_2/d_1) = (t_2/t_1)$
   (b) $(d_2/d_1) = \sqrt{t_2/t_1}$
   (c) $(d_2/d_1) = (t_2/t_1)^2$
   (d) $(d_2/d_1) = (t_1/t_2)^2$

7. Once he had a clear mathematical description for uniformly accelerated motion, Galileo asks: are there any motions in Nature that are like this? He speculates that falling objects might move with uniform acceleration. To test this idea he wants to determine distances and times of fall, but the primitive clocks of the day will not allow him to accurately time fast motions like those of a falling object. So instead he proposes to study balls rolling down ramps to see if they exhibit this kind of motion. To simulate Galileo's experiments run the `InclinedPlane` program. The simulation shows a red ball rolling down a brown inclined plane. Pause the simulation at $t = 1$ s, 2 s, and 3 s. Record the distance the ball has traveled at each of these three times in the table below. Don't worry if you can't stop it at just the right time - Galileo also had a lot of uncertainty in his time measurements. Also calculate the ratios (d/d_1) and $(t/t_1)^2$, where d_1 is the distance at $t_1 = 1$ s.

time (s)	distance (m)	d/d_1	$(t/t_1)^2$
1		1	1
2			
3			

8. Does this data indicate that the ball rolled down the plane with a constant acceleration?

9. Now pause the simulation, change the inclination of the plane to 5°, and click Initialize. Play the simulation and fill in the table below.

time (s)	distance (m)	d/d_1	$(t/t_1)^2$
1		1	1
2			
3			
4			

10. Does this data indicate that the ball rolled down the plane with a constant acceleration?

11. How does the value of a for the shallower plane compare to the value of a for the steeper plane?

12. Galileo felt that his experiments proved that balls rolling down inclined planes move with a constant acceleration regardless of the angle of tilt (although the actual value of the acceleration depends on the tilt). He also found that for a given tilt, the value of the acceleration did not seem to depend on the weight of the ball. He then imagined what would happen if the plane was tilted to 90°, so that it was vertical. Based on the results of the inclined plane experiment how do you think the ball would move as it fell?

13. After all this Galileo comes to the following conclusion: all objects fall with the same constant acceleration. He recognized, however, that this would imply that any two objects dropped from the same height should hit the ground at the same time. How, then, to account for the "hands breadth" difference mentioned above? Galileo blames this on air resistance. He has something very interesting to say about this:

    > As to speed, the greater this is, the greater will be the opposition made to it by the air, which will also impede bodies the more, the less heavy they are. Thus the falling heavy thing ought to go on accelerating in the squared ratio of the duration of its motion [i.e. $d = (1/2)at^2$]; yet, however heavy the moveable may be, when it falls through very great heights the impediment of air will take away the power of increasing its speed further, and will reduce it to uniform and equable motion [i.e. $d = vt$]. And this equilibration will occur more quickly and at lesser heights as the moveable shall be less heavy.

    According to Galileo's statement, which object will be more greatly affected by air resistance?

    (a) a cannonball
    (b) a small rock
    (c) a feather

14. According to the quote from Galileo, if we let an object fall long enough it will actually stop accelerating and start moving with constant speed. This speed is now referred to as the *terminal speed*. Which object will reach its terminal speed the fastest?

    (a) a cannonball
    (b) a small rock
    (c) a feather

15. Which object will have the smallest terminal speed?
    (a) a cannonball
    (b) a small rock
    (c) a feather
16. Using Galileo's idea about air resistance, explain why a heavy object will land a little bit before a light object if both are dropped from the same height.

It is interesting to note that Galileo finds that in a certain situation an object *can* fall with a constant speed, but only after it has accelerated to its terminal speed. In what other circumstances might an object move with a constant speed? We'll explore that next time.