AST 120 Activity 26
Universal Gravitation

Name	Full	Partial	None

In the last two activities we discussed several key results from Book 1 of the *Principia.* In Book 3 Newton applies his new physics to a variety of phenomena in the natural world. But before he gets to that he lays out some ground rules for doing "natural philosophy":

Rule 1: No more causes of natural things should be admitted than are both true and sufficient to explain their phenomena.

Rule 2: Therefore, the causes assigned to natural effects of the same kind must be, so far as possible, the same.

Rule 3: Those qualities of bodies that cannot be increased or diminished and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally.

Rule 4: In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exception.

Newton then proceeds to show that the moons of Jupiter and Saturn follow Kepler's laws in their orbits about those planets. He has already shown that Kepler's laws for the planets imply that they are attracted to the Sun by an inverse-square force, so it must be that the moons of Jupiter and Saturn are attracted to those planets by a similar inverse-square force. He similarly concludes that the Moon is attracted to the Earth by an inverse-square force. This conclusion leads to a startling discovery as we will now see.

1. We didn't make this clear in the last activity (because Newton doesn't make it clear in the *Principia*) but what we actually showed is that the *acceleration* of the planets is inversely proportional to their distance from the Sun ($a \propto 1/R^2$). Similarly, it is the acceleration of an object moving in a circle at constant speed that is given by $a = v^2/R$. Of course, the forces will also be proportional to the same quantities since $F = ma$. Let's see if we can apply these ideas to the motion of the Moon around the Earth. First of all, does the Moon accelerate? Explain your answer.

2. Let's assume the Moon's orbit is circular. Let's try to find the speed of the Moon as it moves around its orbit. We know from Activity 5 that the period of the Moon's orbit is 27.5 days. In the space below convert this into seconds.

$$T_m = 27.5 \text{ days} = \underline{\hspace{4cm}} \text{ hours} = \underline{\hspace{8cm}} \text{ seconds}$$

3. We also know from our parallax measurements in Lab 5 that the Moon is about 239,000 miles (or 385,000 km) from the center of the Earth. Convert this distance to meters and determine the circumference of the Moon's orbit (in m) below.

4. What is the speed of the Moon in its orbit (in m/s)?

5. Use this speed, along with the distance from the Earth to the Moon given above, to find the centripetal acceleration (in m/s^2) of the Moon.

6. Now recall that Newton has already concluded that the Moon's acceleration is caused by a force that attracts it to Earth and that this is an inverse-square force. So the acceleration should also be inversely proportional to the distance of the Moon from the Earth. Note that the radius of the Earth (which we found in Lab 4) is about 4000 miles, or about $1/60^{th}$ the distance from the center of Earth to the Moon. So what would be the Moon's acceleration if it were brought right to the surface of the Earth?

7. In the "Aristotle versus Galileo" lab we determined that objects fall to the Earth with a constant acceleration of about 9.8 m/s^2. How does this acceleration compare to the result you calculated in the previous question?

8. In the *Principia* Newton describes his logic by imagining what might happen if the Earth had several small moons, instead of just one big one. He states that these moons would follow Kepler's laws. Then he says:

   > And if the lowest of them were small and nearly touched the tops of the highest mountains, its centripetal force, by which it would be kept in its orbit, would (by the preceding computation) be very nearly equal to the gravities of bodies on the tops of those mountains. And this centripetal force would cause this little moon, if it were deprived of all the motion with which it proceeds in its orbit, to descend to the earth . . . and to do so with the same velocity with which heavy bodies fall on the tops of those mountains, because the forces with which they descend are equal. . . . Therefore, since both forces - namely, those of heavy bodies and those of the moons - are directed toward the center of the earth and are similar to each other and equal, they will (by rules 1 and 2) have the same cause. And therefore that force by which the moon is kept in its orbit is the very one that we generally call gravity.

   In a later work Newton illustrates this idea by imagining a powerful cannon fired from the top of an incredibly high mountain.[1] He claims that if the cannonball is fired with enough speed it can actually go into orbit around the Earth just like the Moon. To see an interactive version of Newton's illustration, run the `NewtonsMountain` program. The background for the simulation is a diagram from Newton's book *The System of the World* (published posthumously in 1728). The simulation shows a cannonball (black dot) as well as the ball's velocity (blue arrow) and acceleration (red arrow). When the cannonball is fired with an initial speed of 2.5 km/s what happens?

9. What happens if it is fired with an initial speed of 6.9 km/s? (Note: you should set the new speed, click Initialize, click Clear, then click Play.)

10. What happens if it is fired with an initial speed of 7.5 km/s?

11. If a cannonball was put into orbit this way, would its motion be any different than that of the Moon? Is it reasonable to conclude that the force keeping the Moon in its orbit is really the same force that pulls cannonballs down to Earth (namely, gravity)?

12. So when we launch an ordinary projectile on Earth, is its path REALLY a parabola? To test this, reset the initial speed of the cannonball to 3 km/s. In the Model Options menu select Let Projectile Pass Through Earth.[2] Click play and watch the path of the cannonball as it passes through the Earth. Is the path of a projectile fired on Earth a parabola? If not, what is it?

[1] In fact, the mountain Newton describes is ridiculously high. It is so high that it pokes well above the Earth's atmosphere and its height is a sizable fraction of the Earth's radius. So this is what might be called a *thought experiment*.

[2] Note that the simulation treats the Earth as a point mass. This is perfectly fine when the projectile is outside of the Earth, but it is not correct when the projectile passes through the Earth. However, it does correctly illustrate the imaginary continuation of the projectile's initial path under the assumption that the mass of Earth is concentrated at C. If you are interested to see what would happen if we properly accounted for the fact that Earth's mass is spread almost uniformly throughout its entire volume you can deselect Treat Earth as Point Mass under the Model Options menu.

13. What other forces must we now classify as gravitational forces?

14. OK, so it is starting to look like *all* forces are gravitational forces. Let's try to get a better handle on gravity. We'll start with the gravitational forces with which we are most familiar: those on objects near Earth's surface. We know that all objects experience the same downward acceleration ($a = g = 9.8$ m/s^2) as a result of Earth's gravity. Since $F = ma$ by Newton's Second Law, what must we conclude about the *gravitational forces* on these objects?

    (a) The force is proportional to the mass of the object.

    (b) The force is inversely proportional to the mass of the object.

    (c) The force does not depend on the mass of the object.

15. But now let's recall Newton's Third Law, which states that bodies exert equal and opposite forces on each other. This means that while the Earth exerts a force on a falling stone, the stone also exerts an equal force on the Earth. Therefore, *both* forces must be proportional to the mass of the stone. But the roles of the stone and the Earth are reversed when we go from one of these forces to the other. What conclusion can we draw from this situation?

    (a) The gravitational force on an object near Earth's surface is proportional to the mass of the Earth.

    (b) The gravitational force on an object near Earth's surface is inversely proportional to the mass of the Earth.

    (c) The gravitational force on an object near Earth's surface does not depend on the mass of the Earth.

16. Let's review the properties of gravity we have found so far:

    - Like all forces, gravity is an interaction between two objects (from Newton's Third Law).
    - Gravity is inversely proportional to the square of the distance between the two objects.
    - Gravity is proportional to the mass of each of the two objects.

    We can summarize all this in the form of an equation (although Newton never did this):

    $$F_g = \frac{Gm_1m_2}{d^2}$$

    where G is just a proportionality constant (now known as the universal constant of gravitation). Suppose two objects exert gravitational forces of magnitude F on each other. If we double the mass of one of the objects the force will be______________.

    (a) $F/4$

    (b) $F/2$

    (c) $2F$

    (d) $4F$

17. Suppose two objects exert gravitational forces of magnitude F on each other. If we double the distance between the objects the force will be______________.

    (a) $F/4$
    (b) $F/2$
    (c) $2F$
    (d) $4F$

18. Suppose two objects exert gravitational forces of magnitude F on each other. If we double the mass of both of the objects the force will be______________.

    (a) $F/4$
    (b) $F/2$
    (c) $2F$
    (d) $4F$

19. Galileo concluded that all objects fall to the Earth with the same *constant* acceleration. This implies that all objects are subject to a constant force as they fall. But Newton's law of gravity indicates that the gravitational force on an object depends on the distance of the object from the center of the Earth. So is the force on a falling object REALLY constant?

20. Explain why the force on a falling object is *approximately* constant. Note: the radius of the Earth is approximately 4000 miles.

21. We know that the Earth and a stone exert gravitational forces on each other. The Earth and the Moon exert gravitational forces on each other. Jupiter and its moons exert gravitational forces on each other. The Sun and the planets exert gravitational forces on each other. Can we reasonably conclude that *all objects* exert gravitational forces on each other? Defend your answer.

22. So do the planets exert gravitational forces on each other?

23. If the planets exert gravitational forces on each other, then is it true that the total force on a planet is directed exactly toward the Sun? Recall that forces are combined using Newton's parallelogram rule.

24. Newton showed in Book 1 of the Principia that only a force directed toward a fixed center can reproduce Kepler's laws. So if universal gravitation is true, do the planets REALLY follow Kepler's laws?

25. As a final question, consider this: we have spent the entire semester trying to prove that the Sun is stationary and the Earth orbits around it (rather than vice versa). But if we accept Newton's universal gravitation, is the Sun REALLY stationary? Explain your answer.