AST 120 Activity 10
Ptolemy's *Syntaxis* (or *Almagest*)

Name	Full	Partial	None

1. After Apollonius' deferent-epicycle model, the next major innovation in Ancient Greek astronomy seems to have been introduced by Hipparchos in the 2^{nd} Century BC. Hipparchos was trying to model the motion of the Sun and he wanted to account for the Sun's varying speed as it moves along the ecliptic. He found that he could do this without upsetting the doctrine of uniform circular motion. All he had to do was move the center of the Sun's circular orbit away from the Earth. This device is known as an *eccentric* (which basically just means off-center). To see how the eccentric works, run the `Eccentric` program. The simulation shows the Sun moving in a circular orbit centered on the eccentric point (labeled E). The Earth is represented by a blue dot. Also shown is the motion of the Sun through the sky as seen from Earth. Does the Sun move at a constant speed along its deferent?

2. Does the line connecting the Earth and the Sun change at a constant rate? In other words, is the apparent motion of the Sun as seen from the Earth uniform?

3. When is the Sun moving fastest and when is it moving slowest across the sky (as seen from the Earth)? You may want to pay attention to the sky view for this question.
   (a) It appears to move fastest when it is farthest from Earth and slowest when it is closest to Earth.
   (b) It appears to move fastest when it is closest to Earth and slowest when it is farthest from Earth.
   (c) It appears to move at the same rate at all times.
   (d) It moves fastest when it is on the line through the Earth and the eccentric point, and slowest when it is perpendicular to this line.

4. We know that the Sun travels fastest in winter (in early January when the sun is in Sagittarius ♐- see Activity 6) and slowest in summer (in early July when the sun is in Gemini ♊). If you were to travel from the Earth to the Sun's eccentric point, you should travel in the direction of which constellation?

5. This simulation greatly exaggerates the Sun's *eccentricity* (the distance from the Earth to the sun's eccentric point, in units of the Earth-Sun distance[1]). So let's try to calculate the actual eccentricity that the Sun should have. If the sun's average angular speed through the stars is ω_{avg} then its maximum angular speed is $\omega_{max} = \omega_{avg}/(1-\epsilon)$, where ϵ is the distance from the earth to the eccentric point (center of the sun's orbit) divided by the radius of the sun's orbit. Similarly $\omega_{min} = \omega_{avg}/(1+\epsilon)$. So we find that

$$\omega_{max}(1-\epsilon) = \omega_{min}(1+\epsilon).$$

   Solve this equation, symbolically, for ϵ.

6. From observation we know that the sun's maximum angular speed is 1.027° per day (when the sun is in Sagittarius ♐) and it's minimum speed is 0.9442° per day (when the sun is in Gemini ♊). Calculate ϵ for the sun.

7. Input this value for the eccentricity into the simulation and then run the simulation. Watch the sky view. Can you tell that the sun is changing speeds? Is the eccentricity of the sun's orbit easily noticeable?

8. Is an eccentric circular orbit the *only* way to achieve this effect of the Sun's varying speed? Or could we do the same thing with the epicycle we studied in the previous activity? Return to the `SuperiorPtolemaic` simulation. Make sure the Use Simplified Orbits box is checked. In the Select Planet menu, select User Defined. In the box that appears, uncheck the Link Planet to Sun box. Set Deferent ω to 1 and Epicycle ω to 0 (so the planet remains in a fixed position on its epicycle). Click Play and watch the simulation. Describe the resulting orbit below.

[1] The Earth-Sun distance is known as an *Astronomical Unit* (AU). The simulation gives the eccentricity in AU.

9. So do we truly need to use an eccentric, or could we use an epicycle to achieve the effect of the Sun's varying speed? Explain your answer.

10. Which of these two methods (eccentric or epicycle) would you choose for reproducing the Sun's varying speed along the Ecliptic? Explain the reasons for your answer.

11. Quit `Eccentric`. Hipparchos' eccentric seems to account fairly well for the motion of the Sun. It can actually help out with the planets as well. But first we need to identify what the problem is before we can see how the eccentric can help solve it. Run the `EpicycleEccentric` program. This simulation shows a superior planet moving in a deferent-epicycle orbit, but the center of the deferent can be shifted away from Earth as in the `Eccentric` program we just used. For now, though, set the eccentricity to zero and play the simulation. Pay close attention to the retrograde loops of the planet. Look carefully at both the size and spacing of the loops. Which of the following best describes what you see?
    (a) The loops are all the same size and the angle between one loop and the next is always the same.
    (b) The loops are all the same size but the angle between one loop and the next changes.
    (c) The loops are different sizes but the angle between one loop and the next is always the same.
    (d) The loops are different sizes and the angle between one loop and the next changes.

12. Does this fit with what we saw in Stellarium? Were the retrograde arcs of Mars and the other planets regular in this way, or did they vary from one retrograde to the next?

13. The Ancient Greek astronomers were aware that the retrograde loops look different each time, and that their spacing was not uniform. Shifting the center of the planet's deferent away from Earth can help us get closer to solving this problem. Change the eccentricity to 0.3 and run the simulation again. From the perspective above the plane of the ecliptic it is clear that ______________.
    (a) the loops are all the same size and evenly spaced
    (b) the loops are all the same size but not evenly spaced
    (c) the loops are not the same size but they are evenly spaced
    (d) the loops are not the same size, nor are they evenly spaced

14. But how do these loops appear *as seen from Earth*? To figure this out you can watch the simulation, but you may also want to think about the following analogy: suppose you see two trees off in the distance. If you get closer to the trees will the trees appear bigger or smaller? Will the trees appear closer together or farther apart? Since the retrograde loops of Mars in this model are not centered on the Earth, some of them are closer to the Earth and some are farther away. You can conclude that ______________.

    (a) the loops closer to Earth will appear larger and more closely spaced
    (b) the loops closer to Earth will appear smaller and more closely spaced
    (c) the loops closer to Earth will appear larger and more widely spaced
    (d) the loops closer to Earth will appear smaller and more widely spaced

15. Quit `EpicycleEccentric`. To really match the observed motion of the planets the Ancient Greeks added one more piece to their model: the equant. As far as we know the equant was invented by Claudius Ptolemy in the 2^{nd} Century AD. Ptolemy put all these pieces (epicycle, eccentric, equant) together (along with a few other wrinkles for Mercury and the Moon) to compile what was essentially the final system of Ancient Greek astronomy. This system, described in Ptolemy's *Mathematical Syntaxis* (more commonly known as the *Almagest*), dominated astronomy for the next 1400 years. Ultimately, it was the equant that would prove the demise of Ptolemy's system. To see what the equant does, run the `Equant` program. At first glance the simulation looks just like the one for the eccentric. But there is a new point labeled (Q) and a new circle. Watch the simulation carefully. Does the planet move along its deferent at a constant speed?

16. Which of the following statements seems to be true for this model?

    (a) The planet moves fastest along its deferent when it is closest to Earth (and thus farthest from Q).
    (b) The planet moves fastest along its deferent when it is farthest from Earth (and thus closest to Q).
    (c) The planet moves along its deferent at a constant speed.

17. Now pay close attention to the cyan line that runs from the point Q toward the planet. Does this line change its angle at a constant rate?

18. This is the key idea of the equant. The planet moves at a constant rate *as seen from the equant point* Q. It does not move at a constant rate on its deferent (i.e. as seen from the eccentric point E). Nor does it move at a constant rate as seen from Earth. Note that E is halfway between Earth and Q (this is called the *bisection of the eccentricity*). In the model with the eccentric and the equant the planet, as seen from Earth, appears to move ______________ when it is closest to Earth.

    (a) even faster than it does in the eccentric-only model
    (b) slower than it does in the eccentric-only model (but still faster than when it is far from Earth)
    (c) at the same speed as in the eccentric-only model

19. Now quit `Equant`. To see how Ptolemy put all of these pieces together to model the motion of Mars, run `SuperiorPtolemaic` again. Uncheck the box to Use Simplified Orbits. This shows Ptolemy's full orbit for Mars with an eccentric, equant, and epicycle. Let the simulation run for a while and pay close attention to the retrograde loops of Mars. Are they all the same size?

20. Are the loops evenly spaced?

21. Is the retrograde motion (and corresponding changes in brightness) still properly correlated with the motion of the Sun?

22. Now let's try to determine how Ptolemy set the periods of the motions along the deferent and epicycle. Check the box to Use Simplified Orbits again. Run the program and pay close attention to where the planet is on its epicycle when it undergoes retrograde motion. When it is in the middle of retrograde the planet is always on the part of its epicycle …
    (a) …that intersects the deferent to the west of the epicycle's center.
    (b) …toward the center of the deferent.
    (c) …away from the center of the deferent.
    (d) …that intersects the deferent to the east fo the epicycle's center.

23. The last question shows that the period between retrogrades will be the period of the epicycle's motion *in relation to the deferent.* [If you aren't sure what I mean by this, ask.] The period between retrogrades is what we have called the planet's ______________________________ period.

24. The zodiacal period of the planet is related to the period of what motion (measured relative to the stars)?

25. Quit `SuperiorPtolemaic` and run `InferiorPtolemaic`. Carefully watch the motion of the planet. Would your answers to the last three questions be different for an inferior planet (Venus, Mercury) than they were for the superior planets (Mars, Jupiter, Saturn)? If so, how?

26. Now you can use Table 1.1 (page 19) to complete the second and third columns (but not the fourth column yet) of the table below. Note that T_{def} is the period of the planet's deferent motion (measured relative to the stars), while T_{erd} is the period of the planet's epicycle motion measured relative to the deferent.

Planet	T_{def} (years)	T_{erd} (years)	T_{epi} (years)
Mercury			
Venus			
Mars			
Jupiter			
Saturn			

27. Ptolemy always thought of the period of the epicycle motion in relation to the deferent. But it will be convenient for us to also consider the period of the epicycle motion as measured relative to the stars. To do this we make use of the forumla:[2]

$$\frac{1}{T_{epi}} = \frac{1}{T_{def}} + \frac{1}{T_{erd}}.$$

Use this formula to complete that last column of the table above.

28. We saw earlier that for a superior planet the epicycle motion was linked to the Sun, while for an inferior planet the deferent motion is linked to the Sun. Explain how these links between the motion of a planet and that of the Sun are apparent in the table above.

29. Now we have a pretty good idea of how Ptolemy constructed orbits for the planets, and how he was able to set their periods, etc, from observational data. But so far we have been looking at 2D models in which the motion of the planet is confined to the plane of the ecliptic (with ecliptic latitude 0°). We know from Stellarium that the planets do not say on the ecliptic, but can be found above or below it. Think about Ptolemy's theory and state two possible ways that Ptolemy could account for variations in a planet's ecliptic latitude. [If you need a hint, ask.]

30. There is just one last thing we need to explore in Ptolemy's theory of the planets. How does Ptolemy determine the sizes of the epicycles and deferents for each planet? In particular, we are interested to see whether the actual size of the epicycle and deferent are important, or is it only the *relative* size of the two circles that is important? Play around with both `SuperiorPtolemaic` and `InferiorPtolemaic`. Select User Defined from the Select Planet menu and change the values of the deferent and epicycle radii. Note that the deferent radius is set to 5 and the epicycle radius to 3.3 (Ptolemy's values for Mars). Change the deferent radius to 10 and run the simulation. Does changing the size of the deferent alter the motions of Mars as seen from Earth?

31. Now set the deferent radius back to 5 and change the epicycle radius to 4. Run the simulation. Does changing the size of the epicycle alter the motions of Mars as seen from Earth?

32. Now set the deferent radius to 10 and the epicycle radius to 6.6. Note that each of these values is double Ptolemy's value for Mars, so we have changed the size of BOTH circles *by the same factor*. Run the simulation. If both circles are increased (or decreased) by the same factor, does this alter the motion of Mars as seen from Earth?

[2] This formula is derived from the fact that the angular velocity of the planet along its epicycle relative to the deferent is equal to its angular velocity along its epicycle relative to the stars minus the angular velocity of the epicycle along the deferent (relative to the stars).