Exploring the Chaotic Pendulum with the *Maxima* CAS

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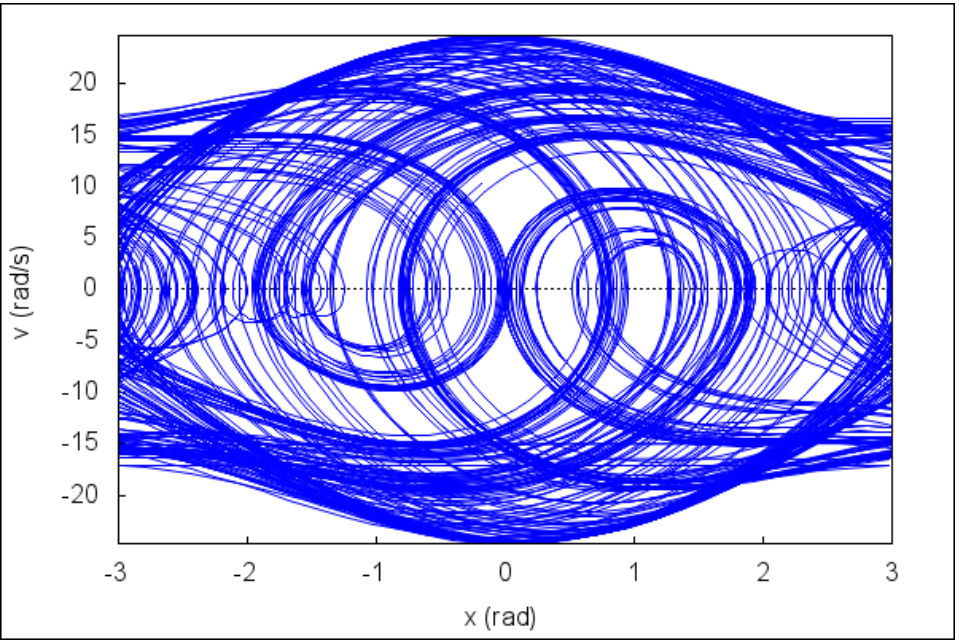
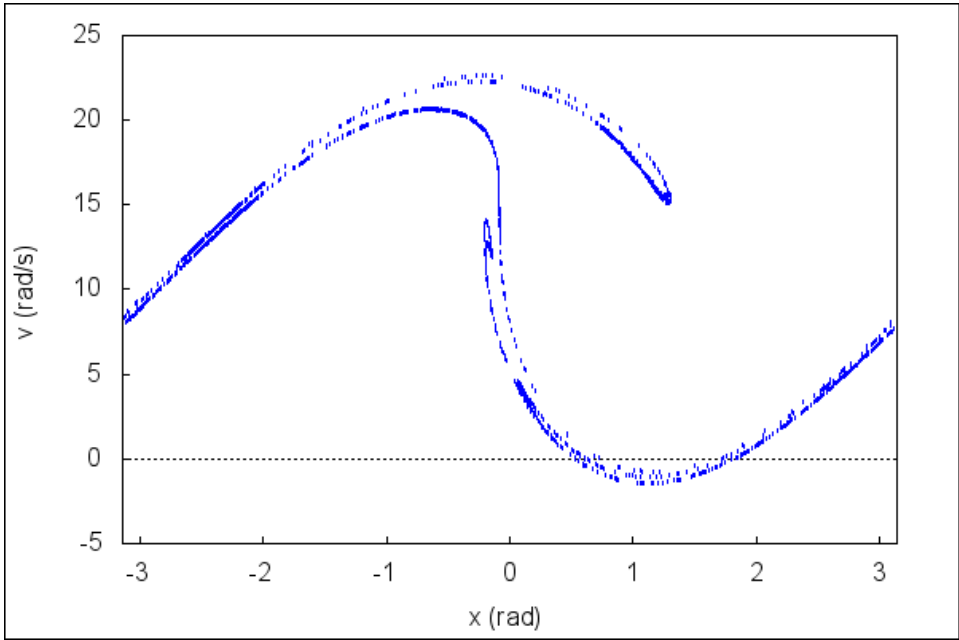




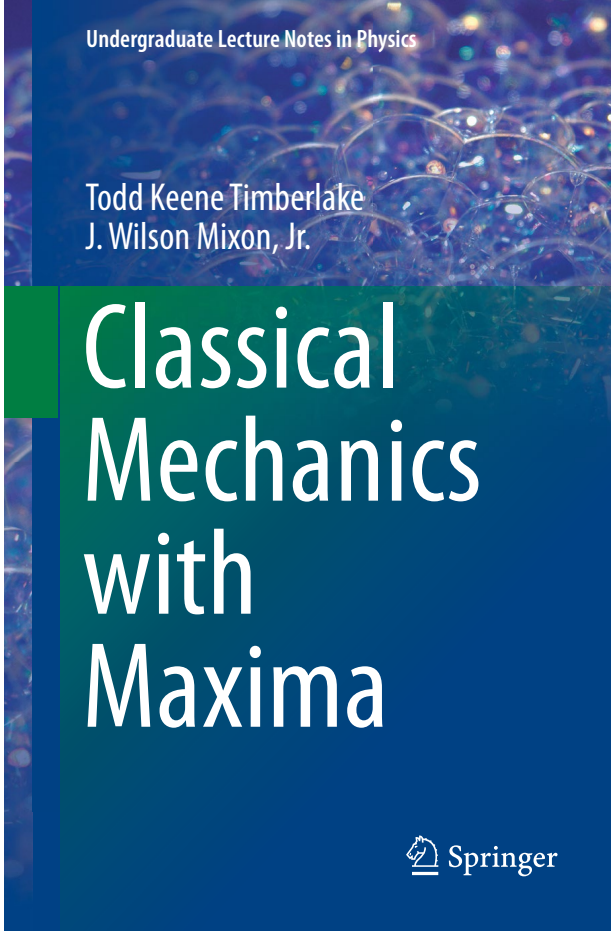
My Course (and book)

- Computational Methods of Physics
- Tutorials and projects with *Maxima* and *EJS*
- All classical mechanics content
- Did similar projects in two-semester classical sequence
- Book site:

sites.berry.edu/ttimberlake/teaching/cm_maxima/

- See review of book in Nov/Dec 2016 *Computing in Science & Engineering*
- Get wxMaxima free at:

wxmaxima.sourceforge.net



Driven, Damped Pendulum

- Ideal pendulum
- Weight:

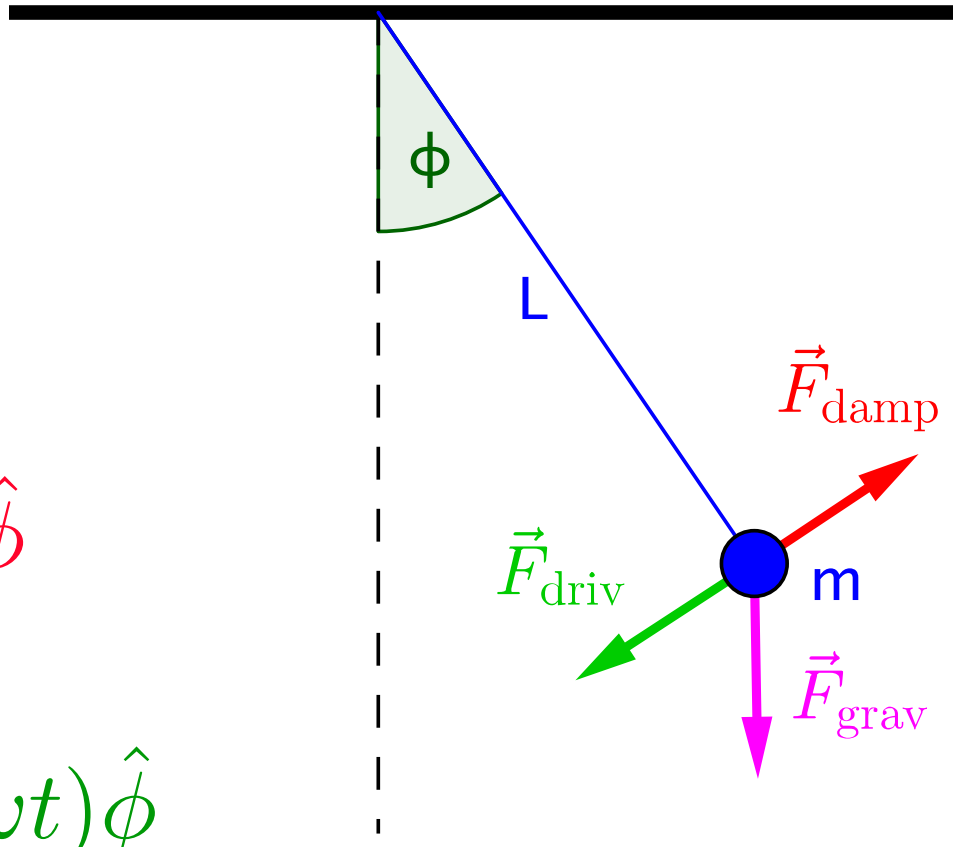
$$\vec{F}_{\rm grav} = -mg\hat{z}$$

- Damping:

$$\vec{F}_{\rm damp} = -bL\dot{\phi}\hat{\phi}$$

- Driving:

$$\vec{F}_{\rm driv} = F_0 \cos(\omega t)\hat{\phi}$$



Equations of Motion

- Torque equation:

$$mL^2\ddot{\phi} = -bL^2\dot{\phi} - mgL\sin\phi + LF_0\cos(\omega t)$$

- New parameters:

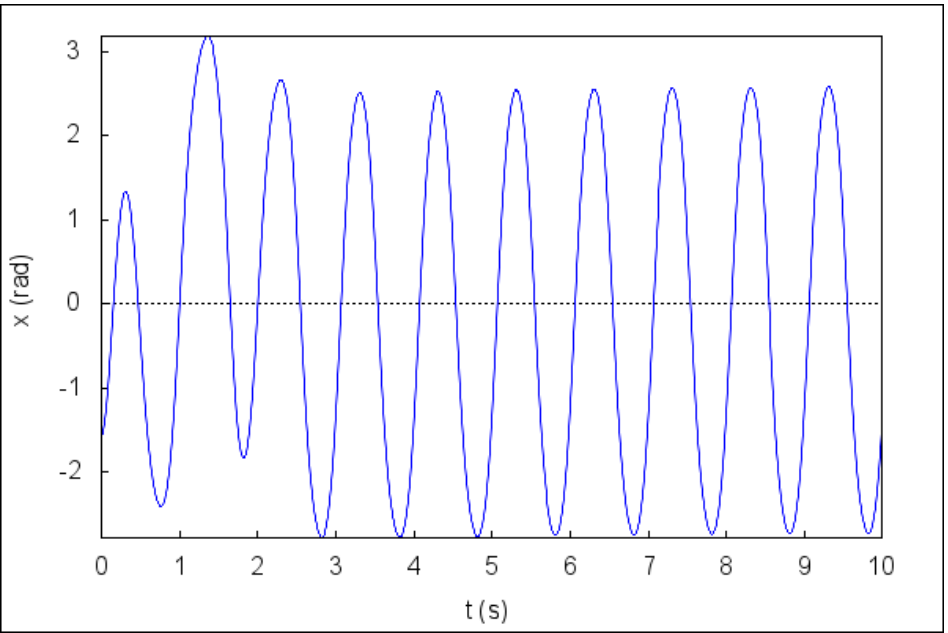
$$x = \phi, \beta = \frac{b}{2m}, \omega_0 = \sqrt{\frac{g}{L}}, \gamma = \frac{F_0}{mg}$$

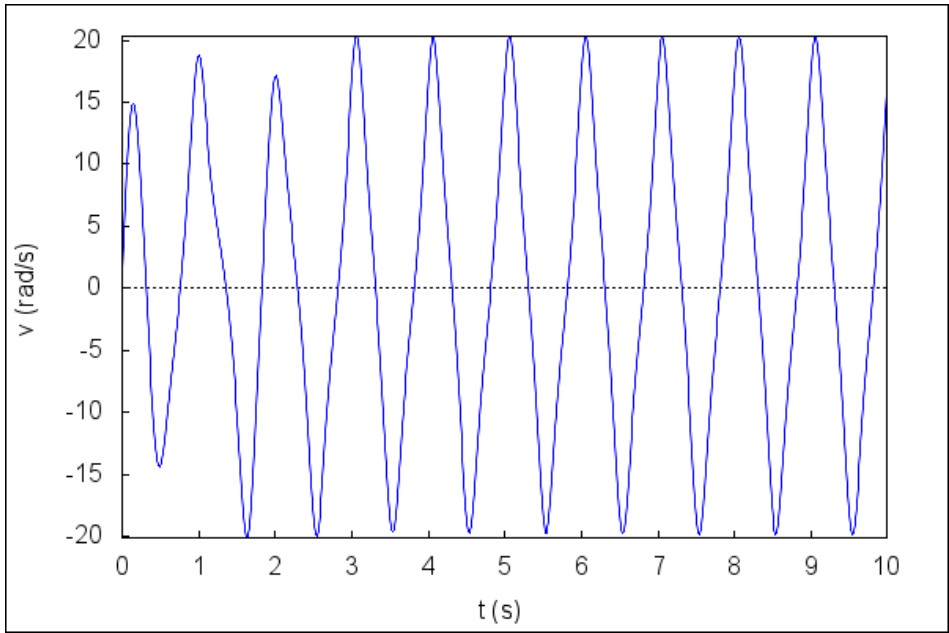
- First-order ODEs:

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= -2\beta v - \omega_0^2 \sin x + \gamma\omega_0^2 \cos(\omega t) \end{aligned}$$

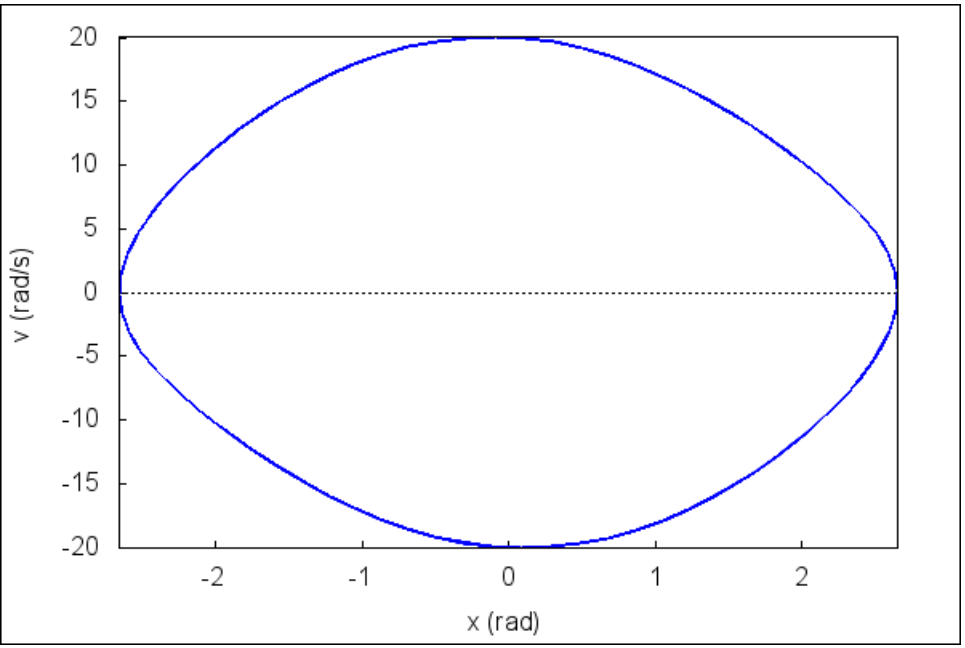
Motion for γ=1

- Initial conditions: $x_0 = -\pi/2, v_0 = 0$
- Solve ODEs using 4th order Runge-Kutta with fixed time step (0.01 s).

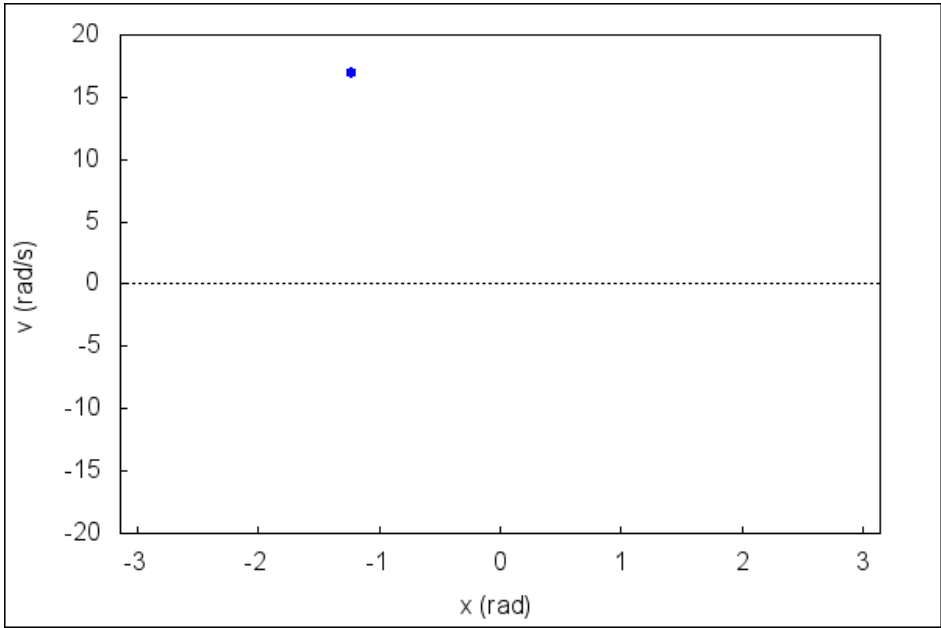




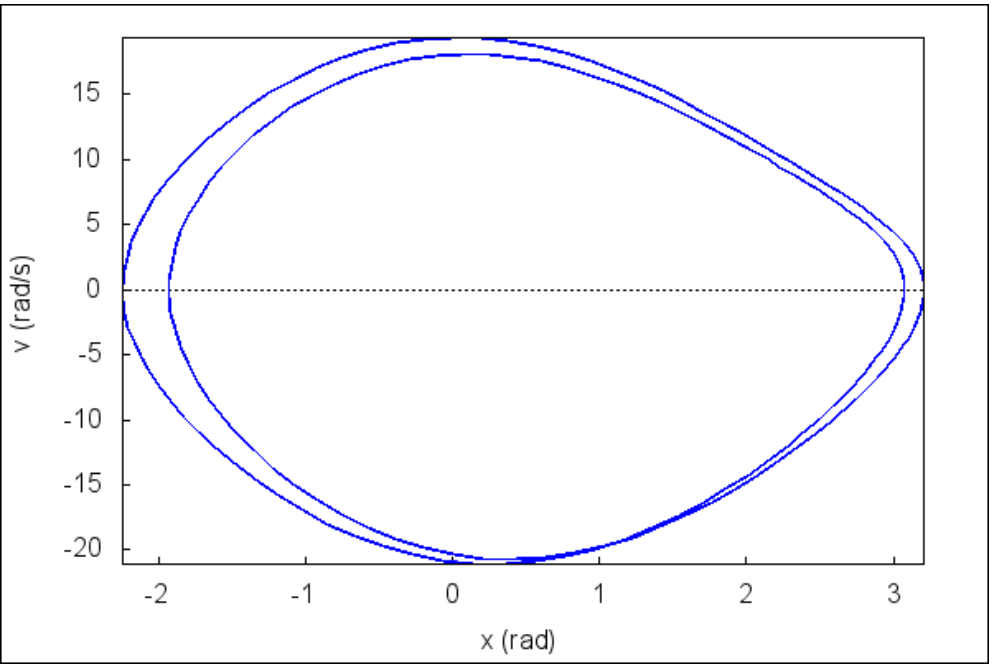
Phase Space and Strobe Plots

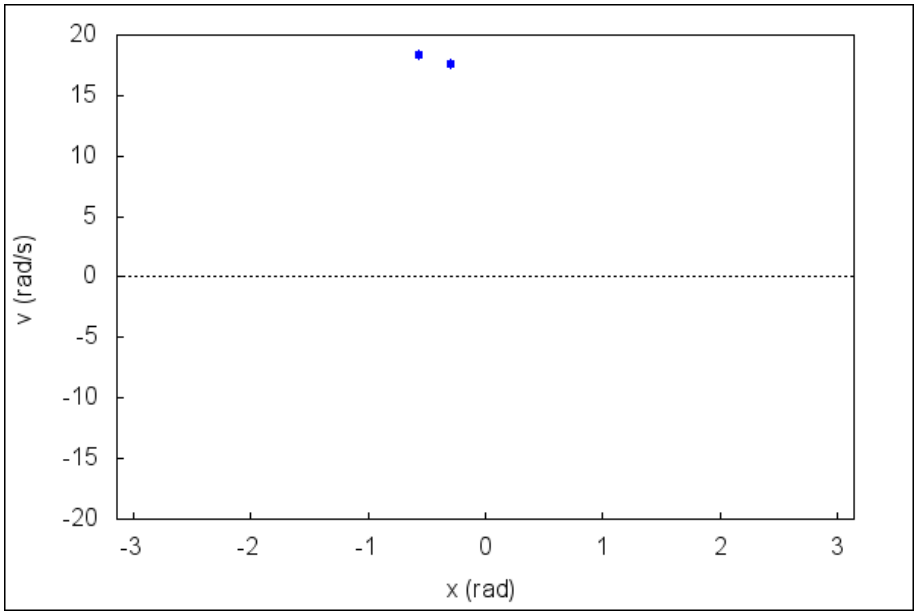


Strobe plot samples location in phase space once per cycle of the driving force, ignoring transient motion.

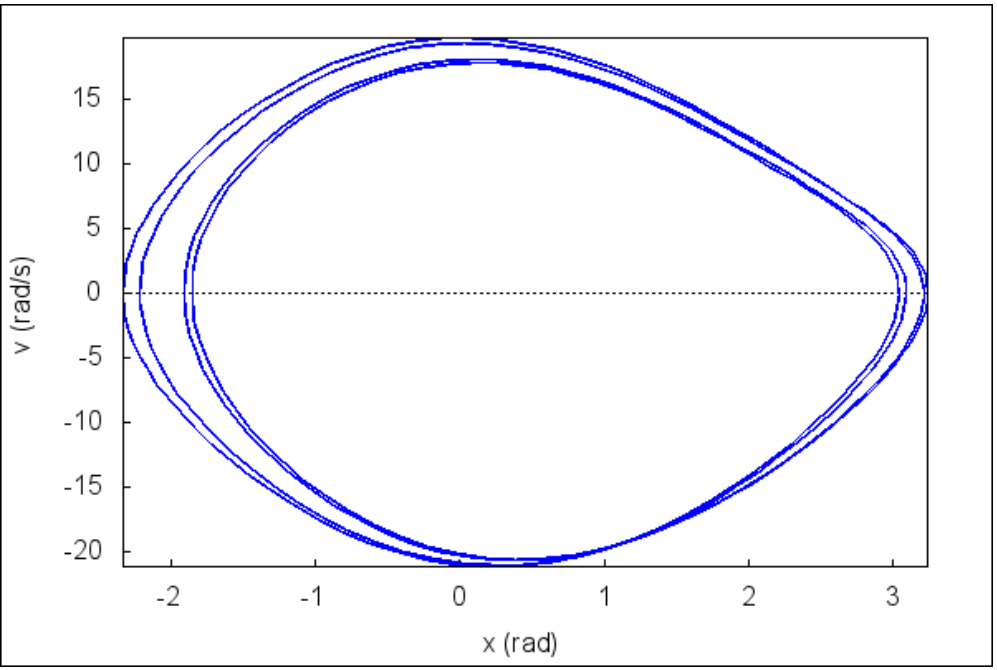


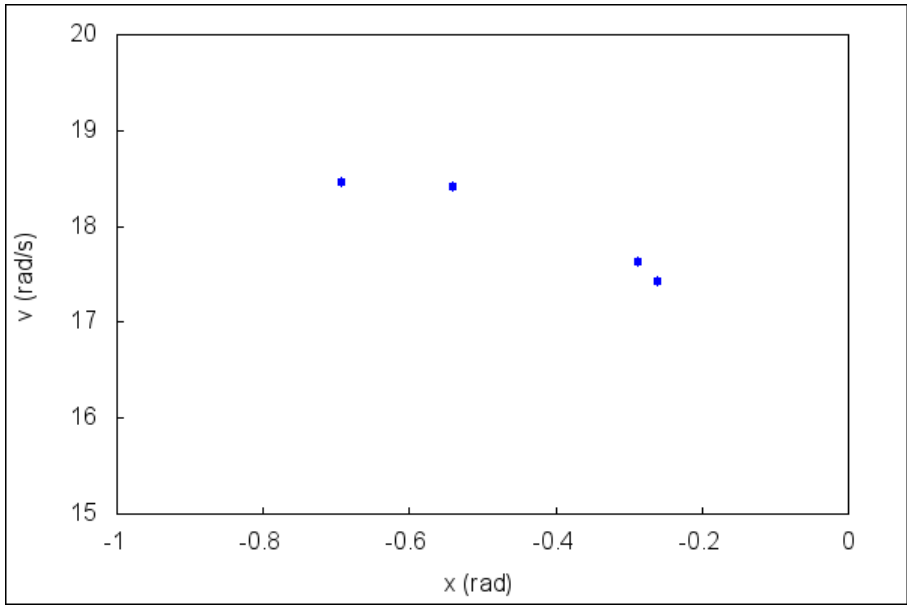
Motion for γ=1.075



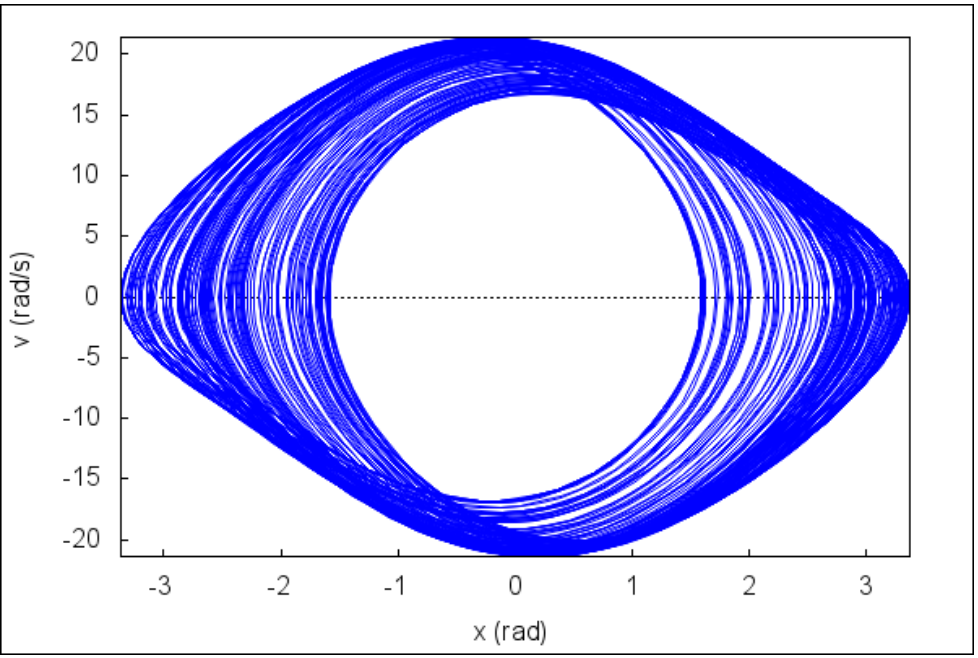


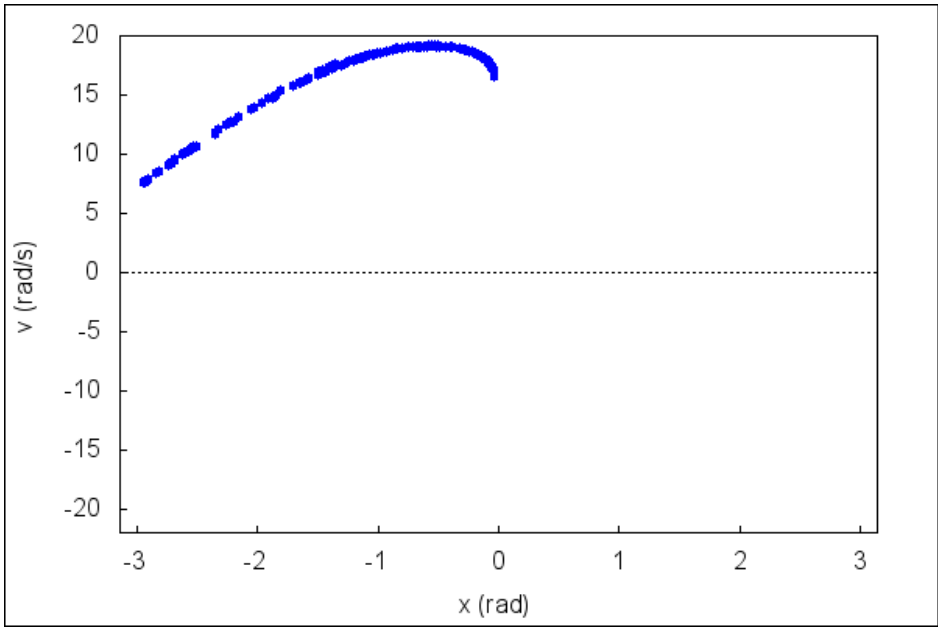
Motion for γ=1.081



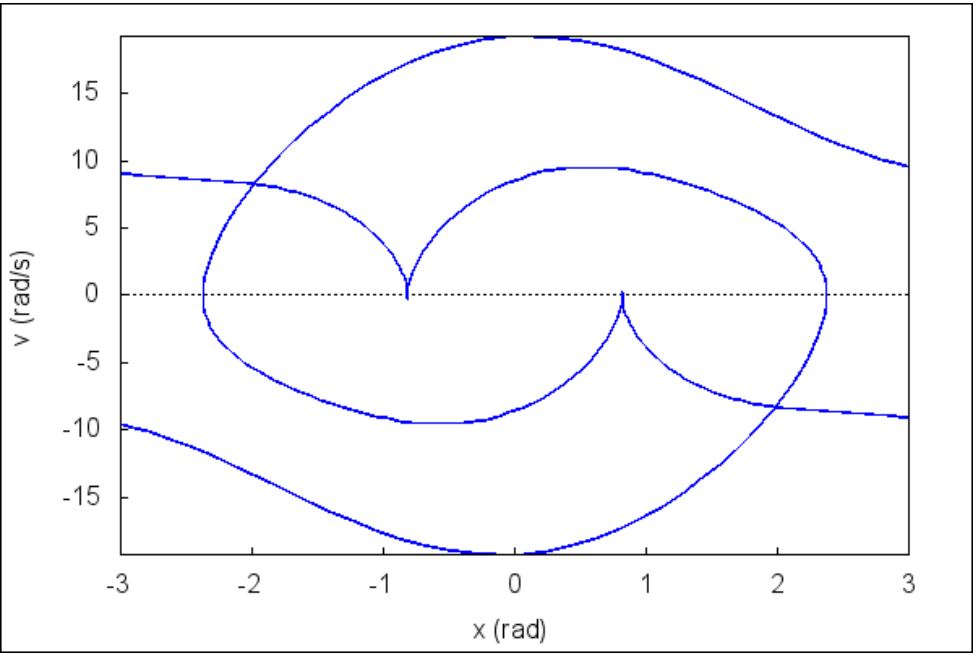


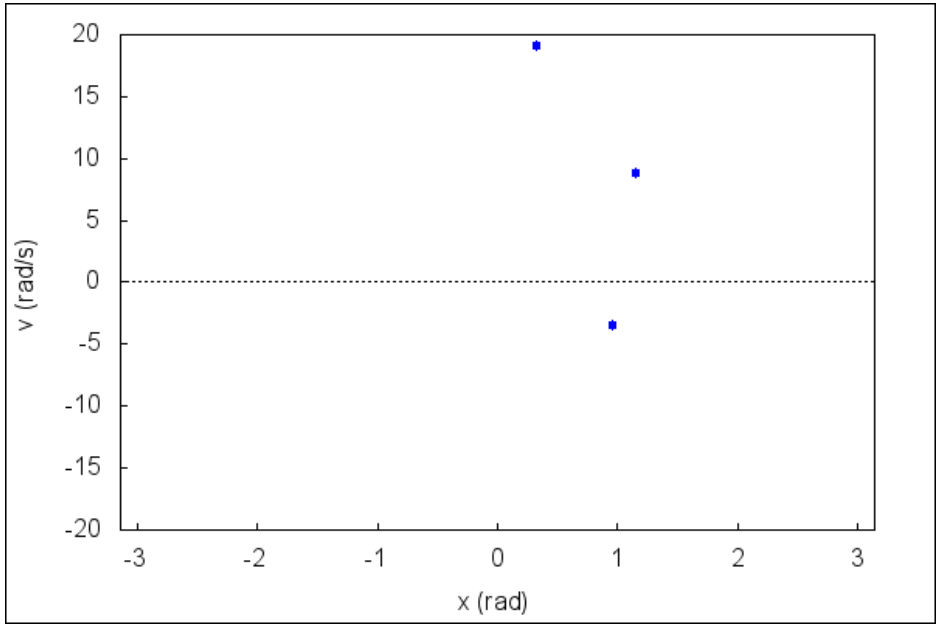
Motion for γ=1.1



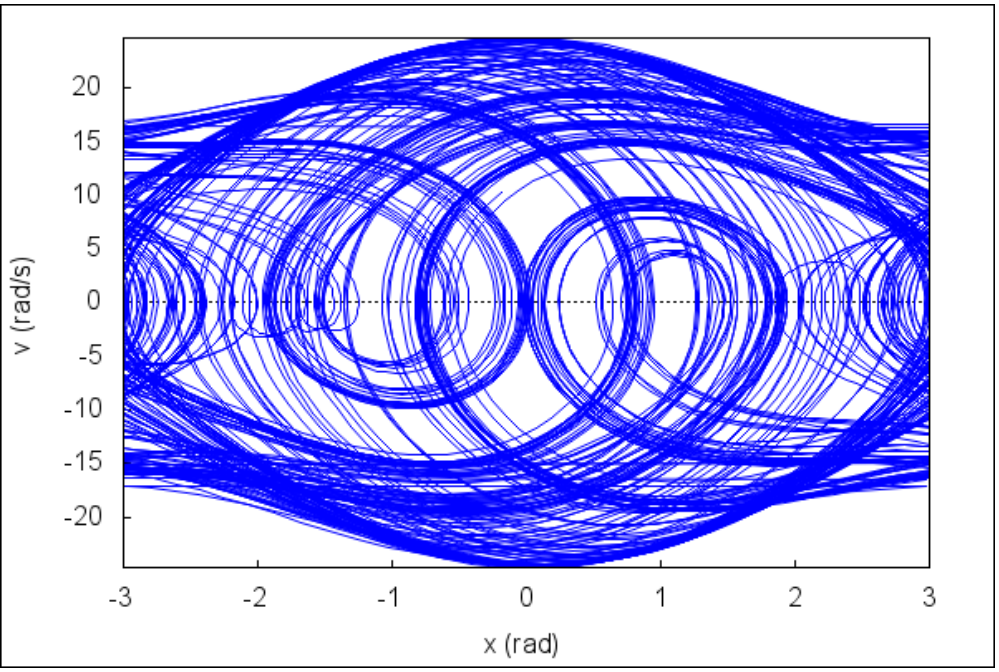


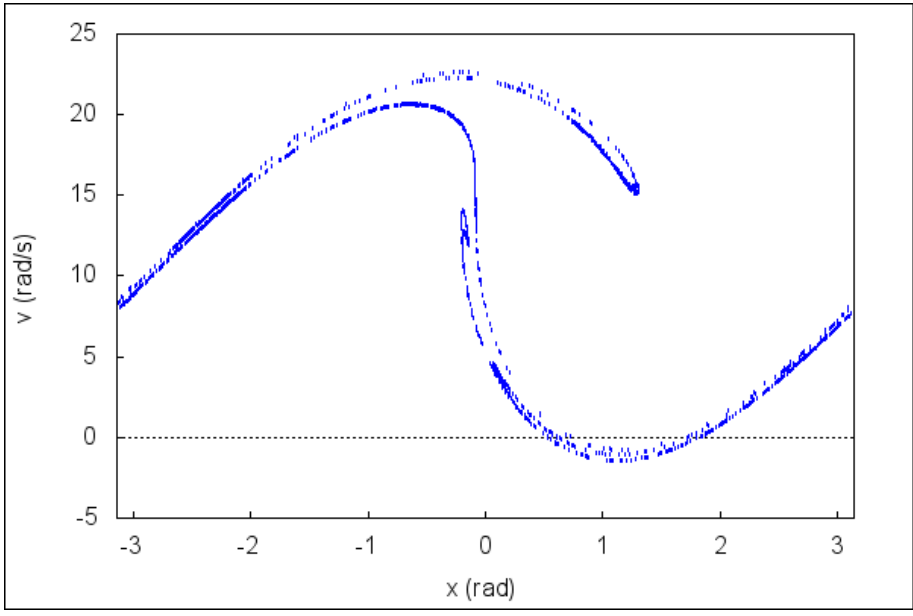
Motion for γ=1.13



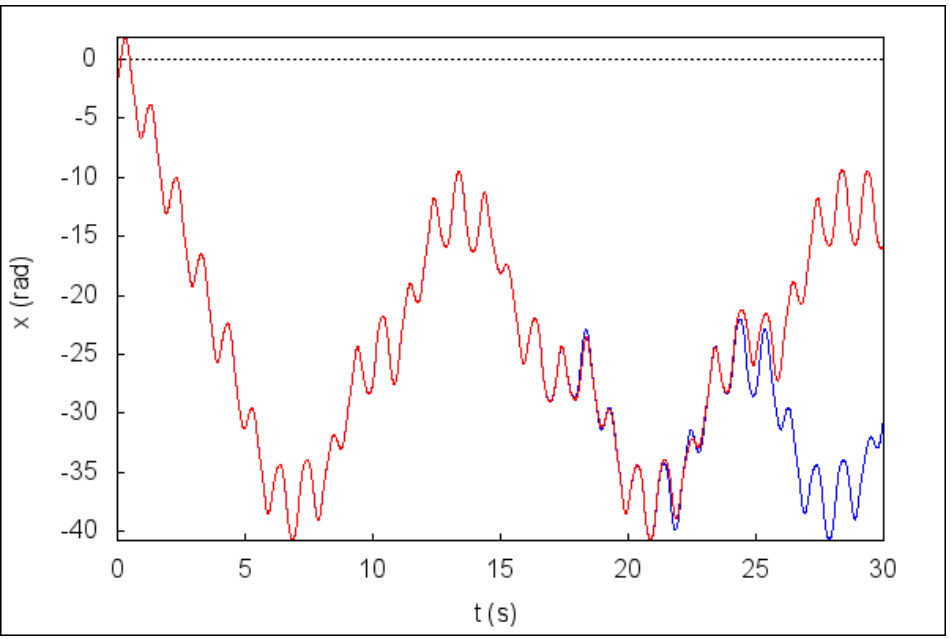


Motion for γ=1.5





Exponential Divergence and Chaos



$$d(t) = |x_1(t) - x_2(t)|$$

$$d(t) \approx d(0)\ e^{\lambda t}$$

$$\ln d(t) \approx \lambda t + \ln d_0$$

$$d_0 = 10^{-6}\ \text{rad}$$