Physics 402 **Hamiltonian Chaos Homework** **Spring 2004**

1 Computational Problems

For these problems you will need to carry out computations on MATHEMATICA. You only need to turn in your plots. It is not necessary to turn in the code you used to create the plots.

1. Create a Poincaré Section for the Standard Map with $K = 1.2$.
2. Create a plot of a chaotic trajectory for the Standard Map with $K = 1.2$. Plot at least 10,000 iterations of the map for whichever initial condition you choose.
3. Illustrate the flow of points inside the period-2 nonlinear resonance by starting with 5000 initial points distributed randomly in a box with corners at $(\theta = 0.45, r = 0.45)$, $(\theta = 0.45, r = 0.55)$, $(\theta = 0.55, r = 0.45)$, and $(\theta = 0.55, r = 0.55)$. Apply the Standard Map with $K = 1.2$ to these points at least four times. Describe the motion of these trajectories.

2 Analytical Problems

Solve each of these problems by hand. Show all of your work.

4 Show that the Henon map (given below) is area-preserving.

$$\begin{aligned} x_{i+1} &= x_i \cos\alpha - (y_i - x_i^2)\sin\alpha \\ y_{i+1} &= x_i \sin\alpha + (y_i - x_i^2)\cos\alpha \end{aligned}$$

Note that α is the nonlinearity parameter for this map. x and y are the dynamical variables of the map.

5 Consider Euler's constant e.

1. Find the first six a_n's (up to a_5) in the continued fraction expansion of e.
2. Find the first six rational approximates of e.
3. Would a KAM torus with winding number e break up sooner or later than a KAM torus with winding number π as the perturbation is increased? Explain your answer.

6 Show that $(0, 0)$ is a fixed point of the Henon map and that it is stable for all values of α.