Electron in Magnetic Field Tutorial

Consider a free electron in a uniform magnetic field $\vec{B} = B_0\hat{k}$. We have shown that the Hamiltonian for this system can be expressed as a matrix in the S_z basis as:

$$H = -\frac{\gamma B_0 \hbar}{2}\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right)$$

where $\gamma \equiv -ge/m_e$ and $g \approx 2$. So the eigenspinors of H are just the eigenspinors of S_z (i.e. $\chi_\pm$). In general the state of the electron will be a superposition of these eigenstates, and this superposition may depend on time. We can express this as a spinor:

$$|\psi\rangle = \left(\begin{array}{c} f(t) \\ g(t) \end{array}\right).$$

This spinor must solve the Schrödinger equation:

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle.$$

1. Write down (ordinary) differential equations for the functions $f(t)$ and $g(t)$.

2. Solve these equations. Assume that $f(0) = \cos(\alpha/2)$ and $g(0) = \sin(\alpha/2)$ (where α is a constant).

3. Find $\langle S_z\rangle$ for this state. Is S_z a conserved quantity?

4. Find $\langle S_y \rangle$ for this state. Is S_y a conserved quantity?

5. Find $\langle S_x \rangle$ for this state. Is S_x a conserved quantity?

6. Describe the behavior of the vector $\langle \vec{S} \rangle$ as a function of time.

7. Find $|\langle \vec{S} \rangle|$. Is this a conserved quantity? How does it compare to $\sqrt{\langle S^2 \rangle} = \sqrt{3\hbar^2/4} = \sqrt{3}\hbar/2$?