The Infinite Square Well

In this tutorial you will solve the EEP for the infinite square well, explore some properties of the energy eigenstates of this system, and visualize various solutions to the Schrödinger equation for this system.

1. Consider a particle moving in 1D and trapped between two hard walls. The potential is

$$V(x) = \begin{cases} 0, & 0 \leq x \leq a \\ \infty, & \text{otherwise.} \end{cases}$$

   We know that the particle cannot exist outside of the hard walls (because it's energy would have to be infinite). So what is the solution $\psi(x)$ for the EEP in this region?

2. Write down the EEP for the region between the walls.

3. Show that $\psi(x) = A\sin(kx) + B\cos(kx)$, where $k \equiv \sqrt{2mE}/\hbar$ is a solution to the EEP between the walls.

4. Refer to the physicality conditions for wave functions. What must be the value of $\psi(0)$? Why? What about $\psi(a)$?

5. Use the conditions given above to find the value of B.

6. Use the conditions given above to find the possible values for k.

7. Explain why we cannot allow $k = 0$ in our solution.

8. How is the solution with $k = -\pi/a$ related to the solution with $k = \pi/a$? Do these really represent distinct physical states for the particle? Do we need to worry about the negative values for k?

9. Use the definition of k to find a formula for the energy eigenvalues E_n.

10. Normalize your solution for the $\psi_n(x)$.

11. Is $\psi(x)$ continuous at $x = 0$ and $x = a$? Is this OK? Why or why not?

12. Is $d\psi/dx$ continusous at $x = 0$ and $x = a$? Is this OK? Why or why not?

13. **Challenge:** If we made the potential energy inside the well V_0, instead of 0, how (if at all) would this change the energy eigenvalues and eigenstates? What is the lowest energy that a particle in this system could have?