Nondegenerate Perturbation Theory Tutorial

- Summary of results for nondegenerate perturbation theory:
  - Energies:
$$E_n \approx E_n^0 + E_n^1 + E_n^2$$
  - Eigenstates:
$$|\psi_n\rangle \approx |\psi_n^0\rangle + \sum_{m\neq n} c_m^{(n)} |\psi_m^0\rangle$$
  - First-order energy correction:
$$E_n^1 = \langle \psi_n^0 | H' | \psi_n^0 \rangle$$
  - First-order eigenstate correction coefficients:
$$c_m^{(n)} = \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0}$$
  - Second-order energy correction:
$$E_n^2 = \sum_{n\neq m} \frac{\left|\langle \psi_m^0 | H' | \psi_n^0 \rangle\right|^2}{E_n^0 - E_m^0}$$

- In this tutorial we will apply these results to an electron in a tilted square well. The Hamiltonian for this system is:
$$H = \begin{cases} \infty, & x \leq 0 \\ \epsilon x, & 0 < x < a \\ \infty, & x \geq a \end{cases}$$
where ϵ is a small, real parameter. Basically this is the normal infinite square well with the perturbation $H' = \epsilon x$ added to the region $0 < x < a$. Recall that the energy eigenvalues of the infinite square well are
$$E_n^0 = \frac{n^2 \pi^2 \hbar^2}{2 m_e a^2}$$
and the energy eigenstates are
$$\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right).$$

- Math tips:
$$\int x \sin^2 x \; dx = \frac{x^2}{4} - \frac{x\sin(2x)}{4} - \frac{\cos(2x)}{8}$$
$$\int x \cos x \; dx = \cos x + x \sin x$$
$$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

1. Find the first-order energy correction for the tilted square well.

2. Calculate the matrix element $\langle \psi_m^0 | H' | \psi_n^0 \rangle$ for this system.

3. Determine the coefficients $c_m^{(n)}$ for the first-order correction to the eigenstates.

4. Write out an approximate expression for the ground state wave function including the three largest terms from the first-order correction.

5. Find the second-order energy correction for the tilted square well. Note that your answer will involve an infinite sum, which you don't have to evaluate.

6. Write out an approximate expression for the ground state ($n = 1$) energy. Include the first order correction and the three largest terms from the second-order correction.