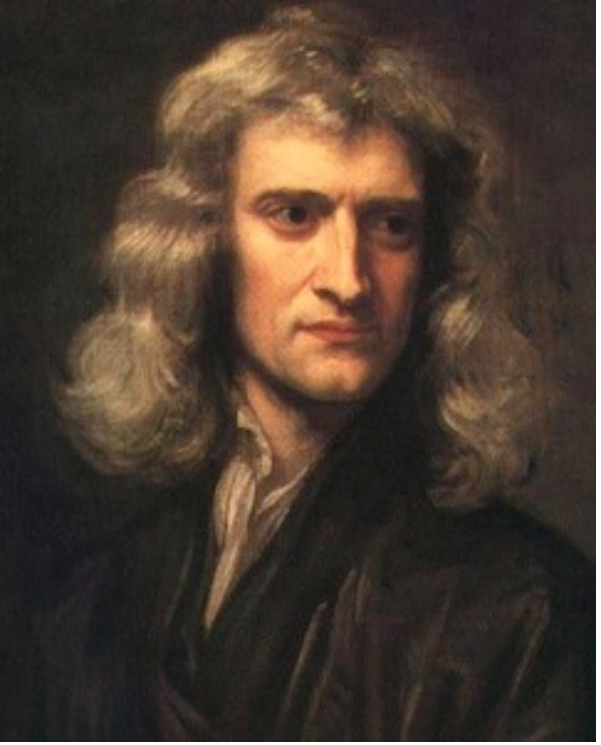


Isaac Newton
by Kneller (1689)

A body falling through the Earth: Newton, Hooke, and the History of Universal Gravitation

Robert Hooke
by Greer (2004)

Todd Timberlake
Vandy Class of '94

BERRY COLLEGE
Physics & Astronomy

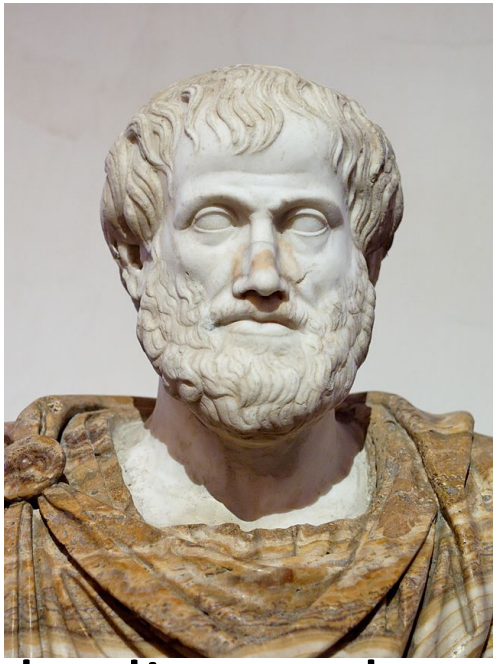
Outline

I. Introduction
II. Historical Background
III. Exchange of Letters Between Hooke and Newton
IV. Aftermath of the Exchange
V. Conclusion

Introduction: The Two Main Issues

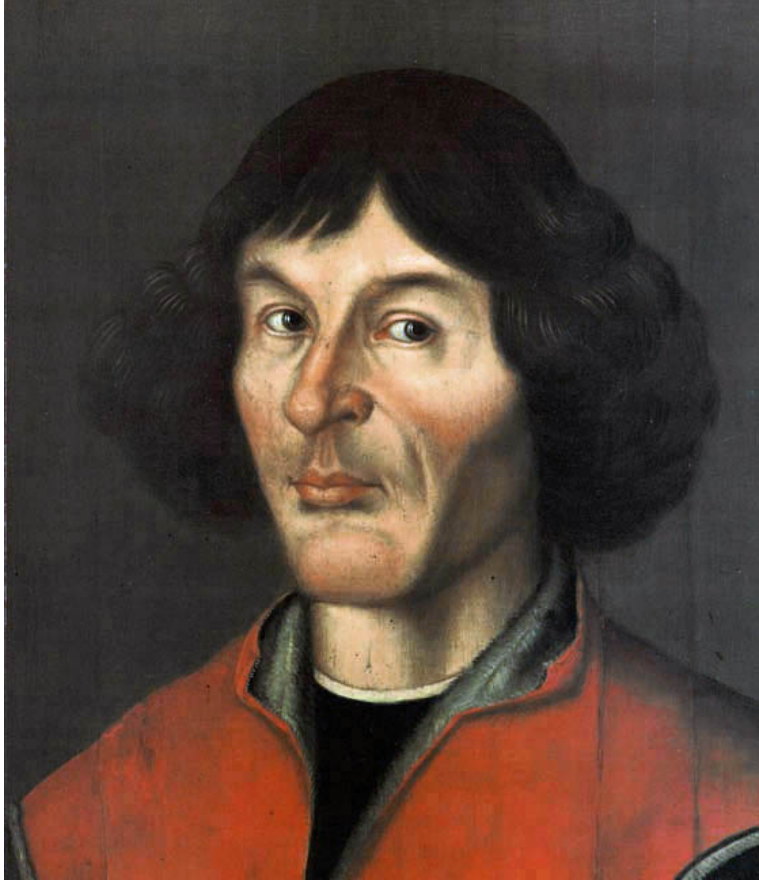
- What do we need to prove Earth's rotation?
  - Theory of inertial motion
  - Theory of falling near Earth's surface
  - Understanding of non-inertial effects
  - Experimental measurement
- What do we need for a theory of universal gravitation?
  - Theory of inertial motion
  - Theory of central forces (attraction)
  - Inverse-square law
  - Universality

Historical Background: Aristotle (4^{th} Century BCE)



- Geometrical theory of gravity: heavy bodies seek the center of the universe (which is, coincidentally, the center of Earth)
- Path of fall is directly toward center, possibly at constant speed (faster for heavier bodies).
- Earth does not rotate. A falling body on a rotating earth would be deflected far to the west (which doesn't happen).

Historical Background: Nicolaus Copernicus (1543)



- Earth rotates once per sidereal day.
- All objects on Earth and in its atmosphere share in this common rotational motion.
- Gravity is an attraction of “cognate bodies.”

Historical Background: William Gilbert (1600)

- In discussing the magnet he proposes that celestial bodies might attract each other through a magnetic force.

Historical Background: Johannes Kepler (1609 & 1621)

- Three Laws of Planetary Motion, but with largely Aristotelian physics
- Thinks sun attracts planets through a magnetic force.
- Area Law viewed as approximation to the Velocity Law (v proportional to 1/r)
- Gravity is a *mutual* attraction of "cognate bodies."
- Thinks a falling body would follow a curved path and land *slightly* west of the point above which it was released.

Historical Background: Francis Bacon (1628)

- Suggests that gravity depends on distance from earth's center: "**For it is very probable, that the Motion of Gravity worketh weakly, both farre from the Earth, and also within the Earth; The former, because the Appetite of Union of Dense Bodies with the Earth, in repsect of the distance, is more dull; The latter, because the Body hath in part attained his Nature, when it is some Depth in the Earth.**"

Historical Background: Galileo Galilei (1632)

- Law of Falling (constant acceleration)
- *Almost* inertia: really circular inertia.
- Suggests that a falling body *might* follow a semicircular path that terminates at Earth's center.
- Later suggests eastward deflection of falling body: "**And far from failing to follow the motion of the earth and necessarily falling behind, it would even go ahead of it, seeing that in its approach toward the earth the rotational motion would have to be made in ever smaller circles, so that if the same speed were conserved in it which it had within the orbit, it ought to run ahead of the whirling earth, as I said.**"
- Ultimately thought Earth's rotation is undetectable.

Historical Background: Pierre Gassendi (1642) and Renee Descartes (1644)

- Gassendi conducts experiments on falling bodies on a ship to contradict Aristotelian arguments about the rotating earth.
- Proposes linear inertia (essentially Newton's First Law), but at times seems to contradict this concept. Descartes uses the idea and sticks with it.

Historical Background: Giambattista Riccioli (1651)

- On eastward deflection: "**If an angel were to let fall a metal sphere of great weight hung to a chain, while holding the other end of the chain immobile, that chain by the force of the sphere might be extended to its full length perpendicularly toward the Earth. But following the Copernicans, it ought to curve obliquely toward the east.**"
- Discusses other effects we would now attribute to the Coriolis force.
- Argues that the absence of these effects proves earth doesn't rotate.

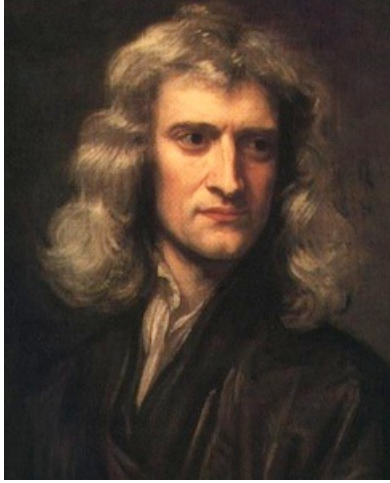
Historical Background: Giovanni Borelli (1667)

- Discusses eastward deflection of a falling body and estimates the deflection as less than an inch for a fall from the (240 foot tall) Asinelli tower in Bologna.
- On a falling body: "**It has to be considered as endowed with two *impetus*, one along the tangent, and the other toward the center of the Earth, and as these *impetus* or corresponding motions, the uniform one along the tangent, and the accelerated one along the radius, do not interfere with each other, the body will move on a curve compounded by both**."
- Proposes that planets are attracted to the sun, but also includes repulsive and sideways forces.

Historical Background: Christiaan Huygens (1673)

- Centrifugal force formula: $F=mv^2/r$.
- Viewed circular motion as a balance between an inward force and the outward centrifugal force.

Historical Background: Isaac Newton (up to 1679)



- Used Kepler's Third Law and Huygen's centrifugal force formula to suggest that sun exerts an inverse-square attraction on the planets.
- He has not yet accepted Kepler's Second Law and sees elliptical orbits as only approximations.
- Shows that Moon's acceleration is related to gravity at earth's surface by inverse-square law, but with 16% error due to bad value for earth's radius.
- Ether theory of gravity, modifies inverse-square law near surface of (and inside) earth.
- Still thinks of orbital motion as balance between attractive and centrifugal forces. Calculates orbits using method of curvature.

Historical Background: Robert Hooke (up to 1679)

- Study of comets and experiments with pendulums and balls rolling on conical surfaces led Hooke to the idea "**that circular motion is compounded of an endeavour by a direct motion by the tangent, and of another endeavour tending to the center**."
- In Micrographia (1665) he states that the globular shapes of the Moon and planets show that the force of gravity exists on those bodies.
- A Statement of Planetary Movements as a Mechanical Problem (1666): Kepler's elliptical orbits can be obtained by compounding inertial motion with an *inflection* toward the Sun due to an attraction.
- In 1666, conducts experiments to see if weight is diminished in a deep well: "**of all parts of the terrestrial globe be magnetical, then a body at a considerable depth, below the surface of the earth, should lose somewhat of its gravitation, or endeavour downwards, by the attraction of the parts of the earth placed above it.**"

- Something like universal gravitation in 1674: "**all celestiall bodys whatsoever have an attraction or a gravitating power towards their own Centers, whereby they attract not only their own parts, & keep them from flying from them, as we may observe the Earth to do, but that they do also attract all the other Celestiall Bodies which are within the sphere of their activity.**"
- This is the first published suggestions that gravity, the force that makes objects fall on earth, also acts between celestial bodies. But it still retains some notion of congruity. He thinks comets may have lost some or all of their gravitating power. Not all objects are subject to gravity.
- Hooke's dynamical law, derived from constant acceleration result ($v^2=v_0^2+2ad$), is that force is proportional to square of speed.
- Combining this with Kepler's velocity law ($v=1/r$) he gets an inverse-square law for the force on the planets. He gets the same law by analogy with the intensity of light.
- Hooke doesn't know how to calculate orbital motions mathematically at this point.

First Letter: Hooke to Newton, 24 Nov 1679

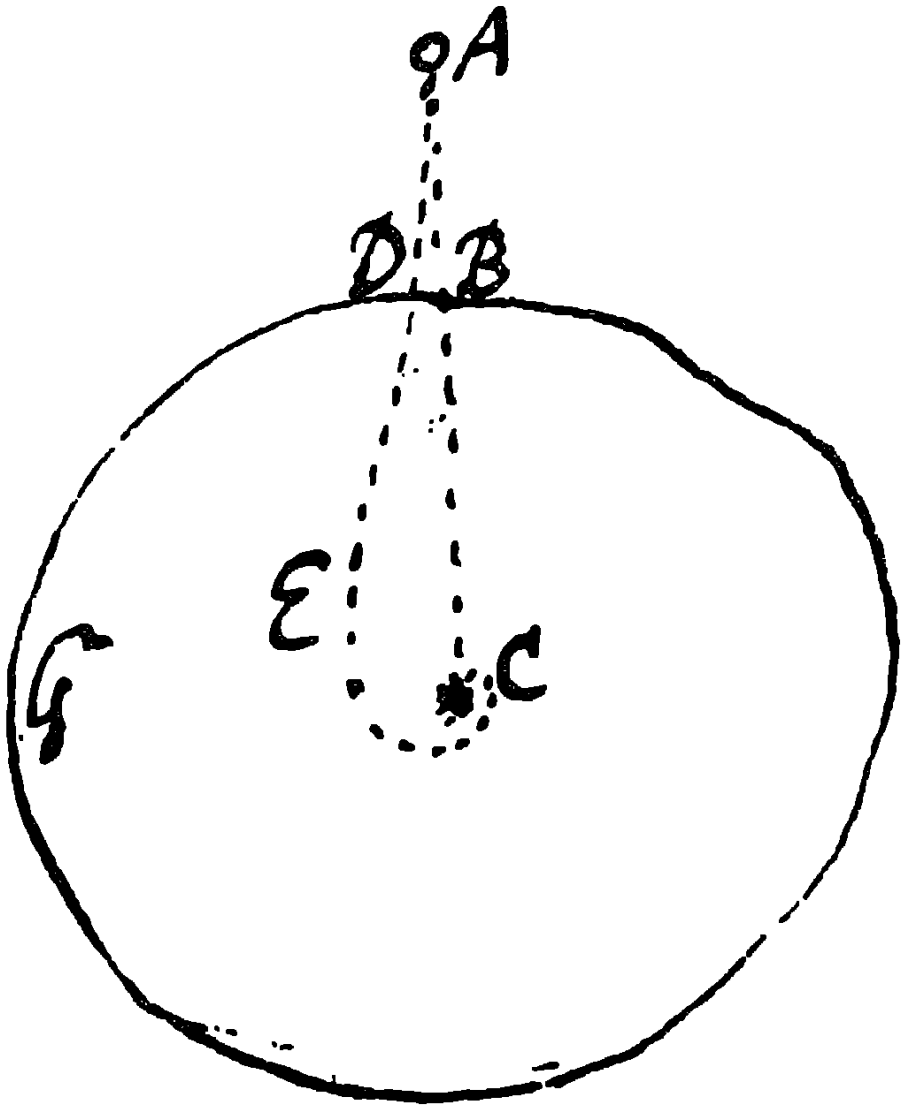
- Hooke is now Secretary to the Royal Society and writes to ask Newton to take part in the Society's discussions.
- Asks for Newton's comments on any of Hooke's hypotheses, especially "**that of compounding the celestiall motions of the planets of a direct motion by the tangent and an attractive motion towards the centrall body…**"
- There is already bad blood between Hooke and Newton because of Hooke's criticism of Newton's optical theories in 1672.

Second Letter: Newton to Hooke, 28 Nov 1679

- Newton declines to attend the Royal Society meetings, saying he is no longer interested in natural philosophy. Claims he has never heard of Hooke's hypothesis.
- He suggests to Hooke that an object dropped from a tall tower would fall slightly to the East, because the object is initially moving eastward (due to Earth's rotation) faster than the point on Earth directly below it.
- Implicit assumptions: object at Equator, no significant forces other than gravity, object released from rest in rotating Earth frame.

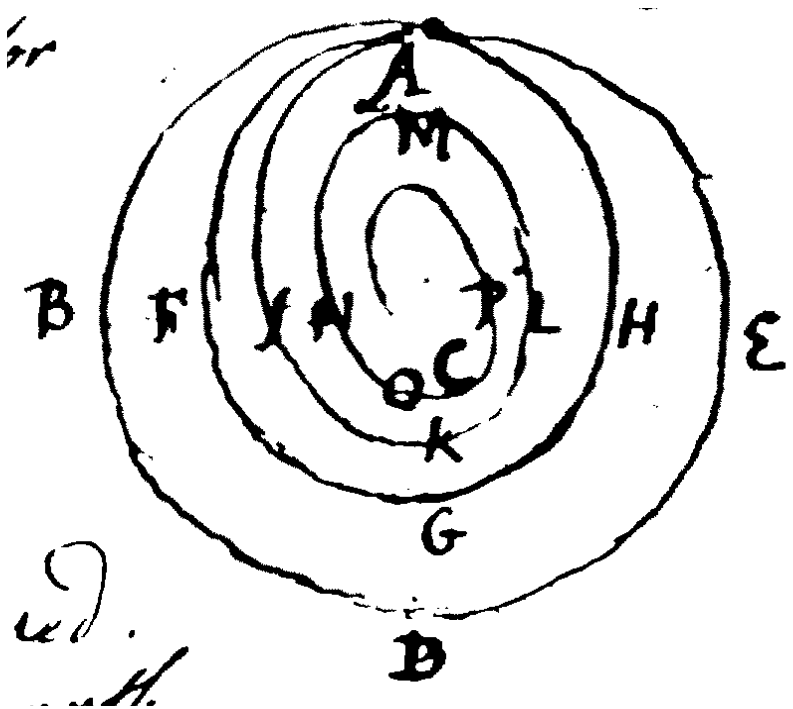
Newton's Sketch

- Newton included a sketch showing the motion of the body to, and *through*, the Earth.
- Note eastward deflection, and spiral path into center of Earth.
- Note: sketch drawn in rotating frame.



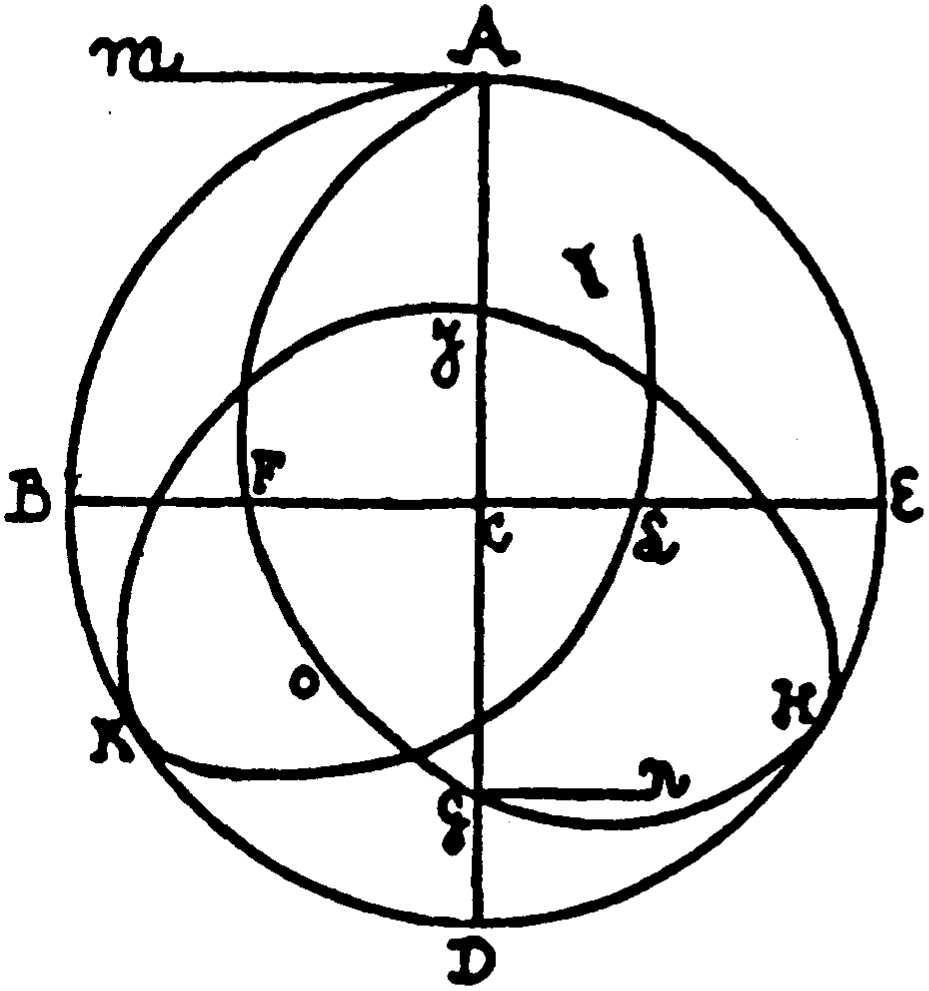
Third Letter: Hooke to Newton, 9 Dec 1679

- Hooke agreed that the body would fall east, but disagreed with the part of Newton's sketch inside Earth.
- He stated that the path would "**resemble an Elleipse**" if the "**gravitation to the former Center remained as before.**"
- Hooke noted that if one includes some resistance within the Earth then the path is an elliptispiral.
- Note: sketches drawn in inertial frame.



Fourth Letter: Newton to Hooke, 13 Dec 1679

- Newton thinks Hooke intends a constant magnitude force inside Earth.
- He responds that the path would not be an ellipse but would precess.
- This sketch drawn in inertial frame.

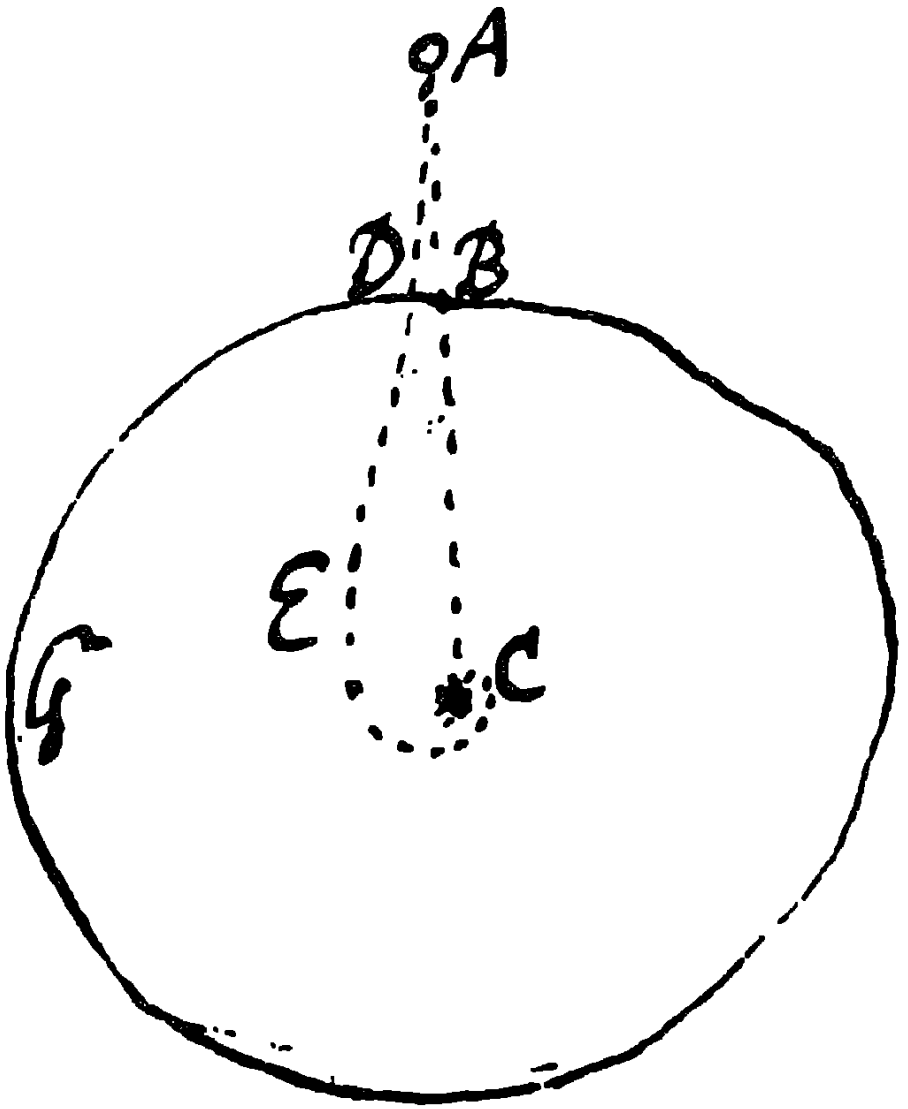


Fifth Letter: Hooke to Newton, 6 Jan 1680

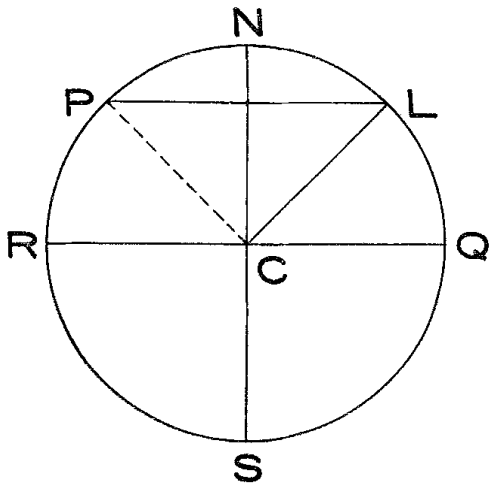
- Hooke explains he meant inverse-square inside Earth: "**my supposition is that the attraction always is in duplicate proportion to the distance from the center reciprocall, and consequently that the velocity will be in a subduplicate (proportion) to the attraction, and consequently as Kepler supposes reciprocall to the distance**"
- Doesn't really think this is correct: "**on the contrary I rather conceive that the more the body approaches the center the lesse will it be urged by the attraction, possibly somewhat like the gravitation on a pendulum …**"

Back to Newton's Sketch

- A uniform density Earth gives a linear Hooke's law force.
- If we include a moderate resistive force, the result is a path very much like Newton's original sketch if we view it in the rotating Earth frame.

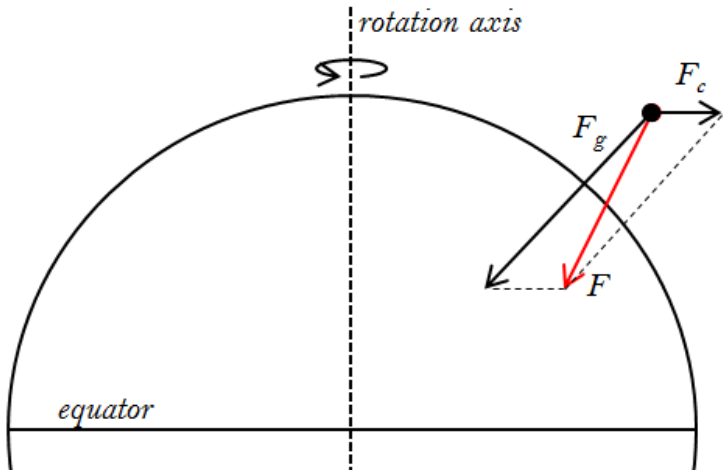


Southeast Deflection



- In his letter of 9 Dec 1679 Hooke claimed that an object dropped at the latitude of London would experience a southward deflection in addition to the eastward deflection.
- "**But if the Body lit fall be not in the aquinochill plain as here at London La 51° 32' *the elleipsed* will be made in a plaine inclined to the plaine of the Equinox 51 32 soe that the fall of the Ball will not be exactly east of the perpendicular but south East and indeed *more to the south then east*..**"
- Newton agrees with Hooke's conclusion.

Which Way is Up?



- If we define up as directly away from Earth's center, Hooke is correct.
- Southward deflection occurs because plane of motion (in inertial frame) does not cut Earth along a latitude line, but along a great circle. In the rotating frame there is a centrifugal force.
- If we define up as opposite the direction of the effective gravitational force (including the centrifugal force) then this southward deflection is almost cancelled out.

Hooke's Measurements

- From 6 Jan 1680 letter to Newton: "**I have (with as much care that I could) made 3 tryalls of the experiment of the falling body, in every of which the ball fell towards the south-east of the perpendicular, and that very considerably, the least being above a quarter of an inch, but because they were not all the same I know now which was true.**"
- From a 17 Jan 1680 letter to Newton: "**I can now assure you that by two tryalls since made in two severall places within doors it succeeded Also. Soe that I am perswaded the Experiment is very certaine, and that It will prove a Demonstration of the Diurnall motion of the earth as you have very happily intimated.**"

Later Measurements

Name	Year	Lat (°)	h (m)	dE (cm)	pred	dS (cm)	pred	# trials
Guglielmini	1791	45.5	78.3	1.9	1.08	1.2	0.00011	16
Benzenberg	1802	36.5	76.3	0.9	0.87	0.34	0.0001	32
Reich	1831	42	158.5	2.8	2.94	0.44	0.00045	106
Rundell	1848	39.5	400	?	11.2	~25	0.00028	~50
Hall	1902	48	23	0.15	0.18	0.005	0.0000095	948
Flammarion	1903	41	68	0.63	0.81	-0.16	0.000083	144

- Eastward deflections of Reich and Hall consistent with predictions. Still no reliable detection of Southward deflection.
- Sources of error: directional alignment, local gravitational anamolies, air currents, motion of plumb line, etc.
- Better way to demonstrate Earth's rotation was introduced by Leon Foucault in 1851.

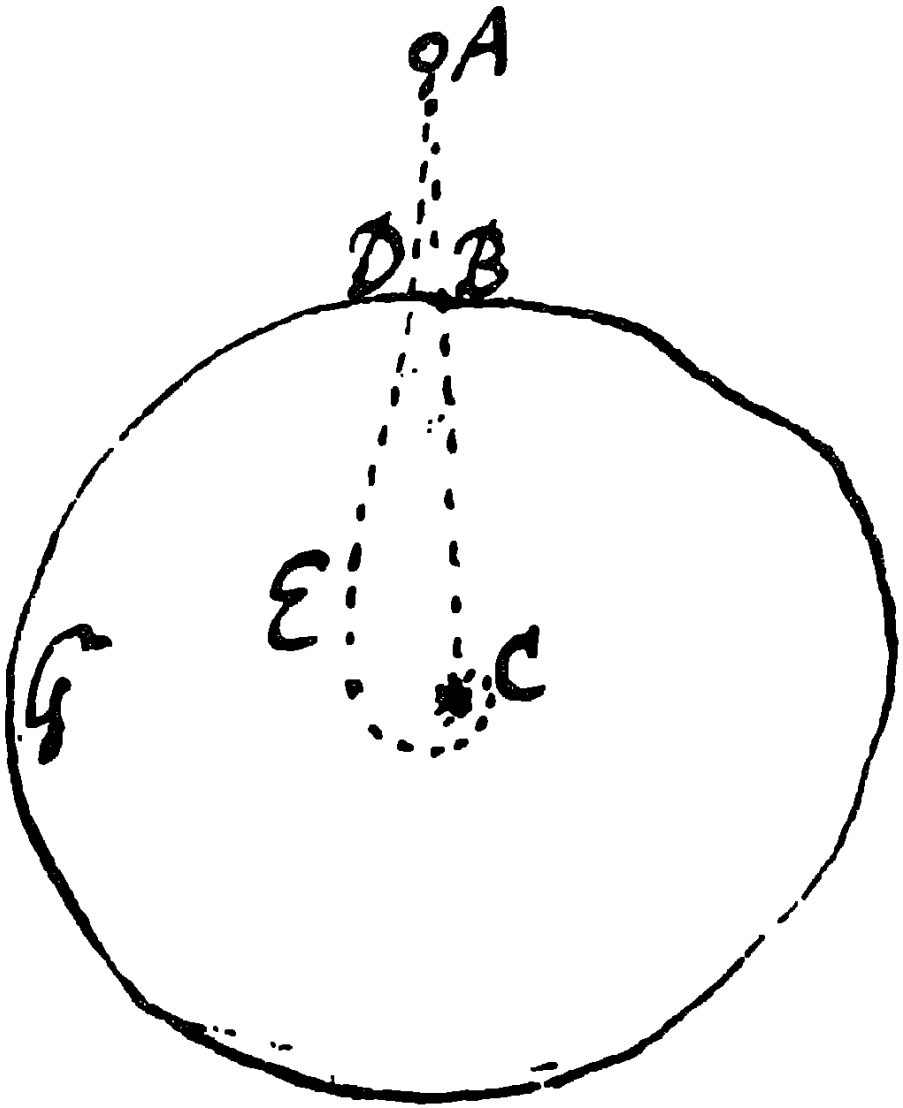
Newton After the Exchange

- Stops using centrifugal force, treats orbital motion as compounded of straight inertia and a central attraction.
- Develops a computational method (much like the Euler method) for finding the path of a body given a force law.
- Shows that inverse square attraction reproduces Kepler orbits, gravity on Earth, etc.
- Publishes the *Principia* in 1687.

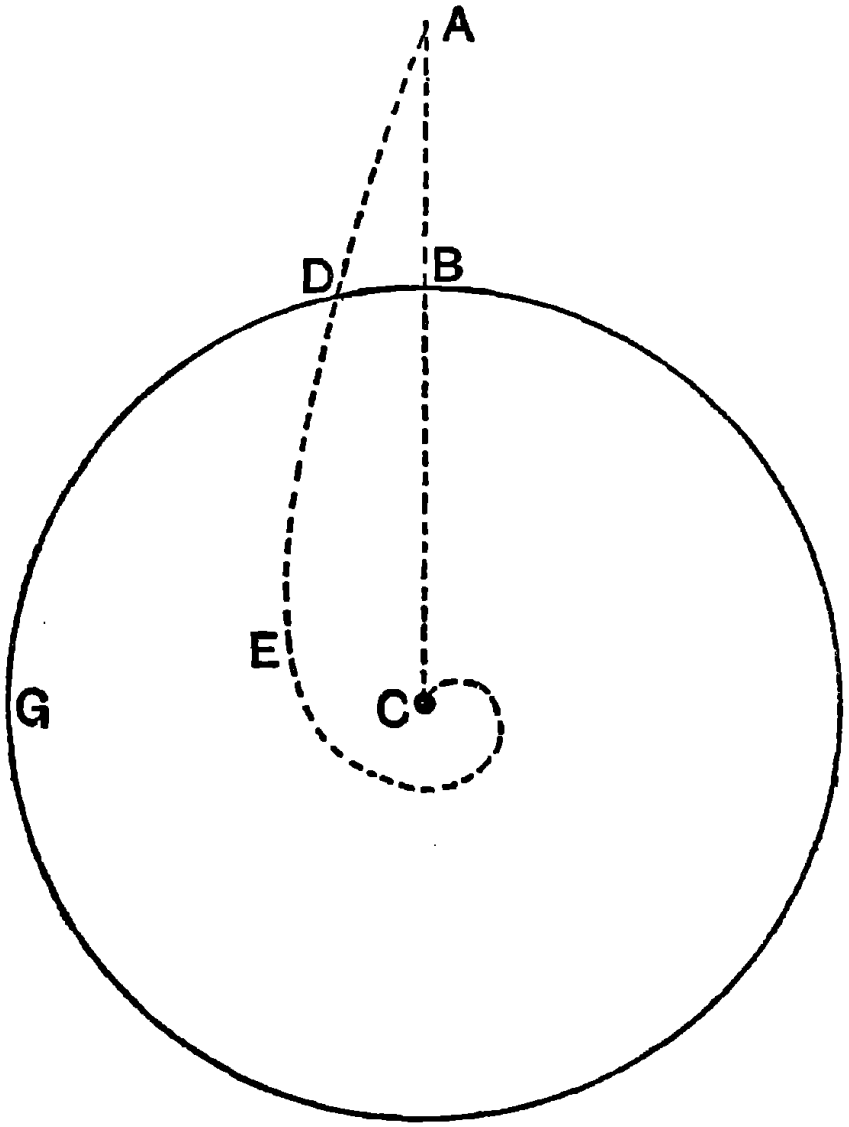
What did Newton learn from Hooke?

- Hooke: the inverse-square law and universal gravitation
- Newton: nothing
- Modern consensus: orbital motion as a compounding of inertia and attraction
- Hooke was the first to properly formulate the problem of relating orbital motion to falling on Earth. But he lacked the mathematical tools to demonstrate the solution, and thus could only argue by analogy.

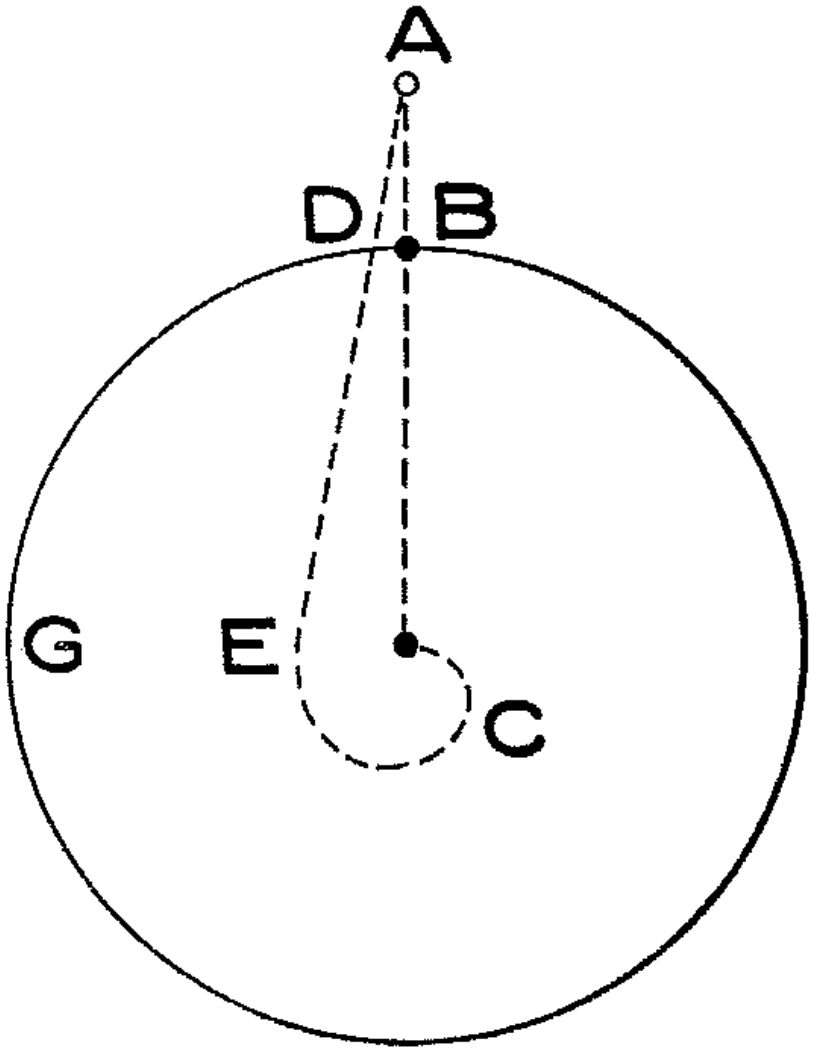
Corruption of Newton's Sketch



Newton 1679



Ball 1893



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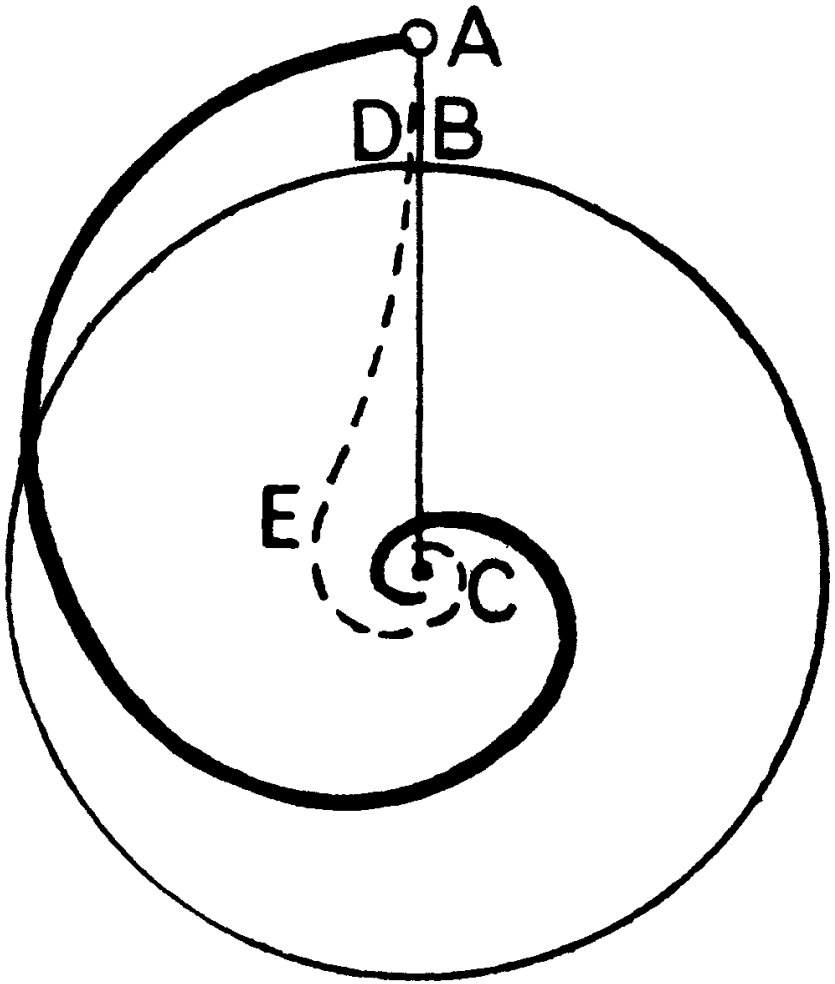


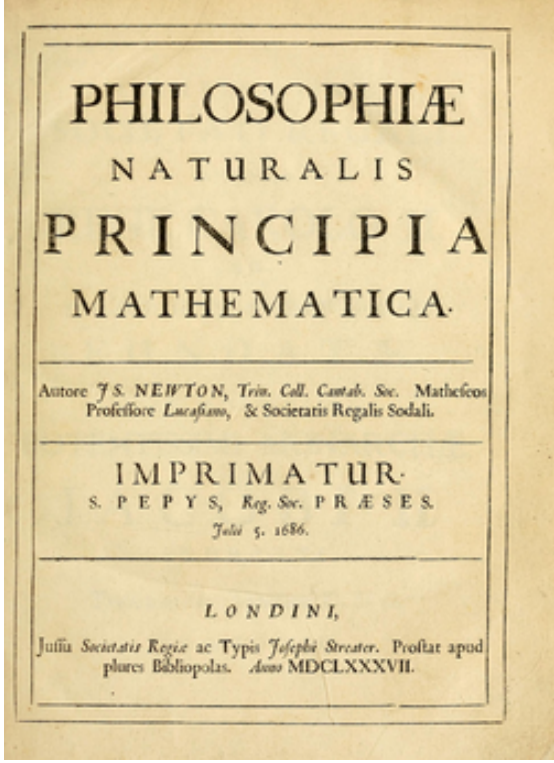
Whiteside 1964

Sources of Confusion in the Story of Universal Gravitation

- Vague ideas about gravity (congruity, ether theory)
- Vague ideas about motion (natural circular motion, balancing centrifugal force with attraction)
- Unstated assumptions about mathematical models and reference frames
- Confusion about the definition of “up”
- Focus on qualitative (rather than quantitative) predictions allows incorrect and vague ideas to survive.

A Turning Point

- The antidote is mathematics: mathematical (or computational) physics requires precise theories, explicit assumptions, and provide quantitative predictions.
- These aspects make physics much more falsifiable (even my simulations!), which drives progress.
- Hooke wanted to produce a mathematical theory, but it was Newton who had the tools to do so. Others had applied mathematics to pieces of physics, but the *Principia* provided an entire mathematical framework for doing physics.



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore Js. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Main References

- J. Lohne, “Hooke versus Newton: An analysis of the documents in the case on free fall and planetary motion,” *Centaurus* **7**, 6 (1960).
- H. Burstyn, “The deflecting force of the earth’s rotation from Galileo to Newton,” *Ann. Sci.* **21**, 47 (2006).
- A. Koyré, “A documentary history of the problem of fall from Kepler to Newton,” *Trans. Am. Phil. Soc.* **45**, 329 (1955).
- R. Westfall, “Hooke and the Law of Universal Gravitation: A reappraisal of a reappraisal,” *Brit. J. His. Sci.* **3**, 245 (1967).
- J. Lohne, “The increasing corruption of Newton’s diagrams,” *His. Sci.* **6**, 69 (1968).
- M. Tiersten and H. Soodak, “Dropped objects and other motions relative to the noninertial earth,” *Am. J. Phys*. **68**, 129 (2000).