Physics 402 - Classical Mechanics II
Research Report

First Draft Due 4/21/04 by 2 PM.
Final Draft Due 4/28/04.

The Project

In this project I want you to explore the dynamics of a Hamiltonian Map other than the Standard Map we have discussed in class. Your group will examine the following map:

$$\Theta_{n+1} = \Theta_n + \cos(2\pi r_{n+1}), \text{ Mod } 1$$
$$r_{n+1} = r_n - \frac{K}{2\pi}\cos(2\pi\Theta_n), \text{ Mod } 1$$

This map displays some features that are very similar to the Standard Map. However, it has some characteristics that are not present in the Standard Map, as you will see. As you explore the dynamics of this map, consider how it is similar to the Standard Map and how it is different. Make sure to include any insights you gain from that comparison in your report.

Tools of the Trade

For your *Mathematica* computations you should follow the examples shown in the *Mathematica* notebook on the Standard Map that I handed out in class (a copy is available on VikingWeb if you need it). Of course, you will have to modify the *Mathematica* code to fit the new map that you are studying. If you are not sure how to make the necessary modifications, then come ask me. I will explain how the code for the Standard Map works so that you will be able to adapt it to your map.

Suggestions

Some things you might want to do:

- create surfaces of section for various parameter values,
- zoom in to show details of the surfaces of section and show fractal structure,
- show examples of regular and chaotic trajectories,
- show that the map is area-preserving (you can do this by hand),
- illustrate graphically that the map is area-preserving,
- locate fixed points (there are at least 8 of them and they can be located analytically as well as graphically),
- determine the parameter values for which each fixed point is stable or unstable (analytically),
- show stable and unstable manifolds (and thus the homoclinic tangle) associated with an unstable fixed point,
- determine the approximate parameter value for which the last KAM torus is destroyed,
- determine the approximate parameter value for which hard chaos occurs.

I'm sure you can think of other things. I realize that you have a finite amount of time to work on this. You don't have to do everything in that list to get a good grade. However, the more you do the better your grade will be (up to a point -- don't be repetitive). And, of course, make sure to leave yourself time to write a good report of your work.

Good luck, and don't hesitate to ask for my help.