The Scale of the Universe

1 Goal of This Activity

In this activity we will use observational data to determine which planets in the Copernican system have orbits inside Earth's orbit and which planets have orbits outside of Earth's orbit. Then we will determine the size of the orbit of each planet compared to Earth's orbit.

2 Useful Facts We Have Already Found

- The angle between a planet and Sun is called the planet's *elongation*. If a planet has an elongation of 0° it is said to be in *conjunction*. If it has an elongation of 180° it is said to be in *opposition*. If it has an elongation of 90° (east or west) it is said to be in (eastern or western) *quadrature*.
- Mercury and Venus are never seen very far from Sun. Mercury has a maximum elongation of about 28°. Venus has a maximum elongation of about 48°.
- Mars, Jupiter, and Saturn can have any elongation (ie they can reach opposition).
- All planets generally move eastward relative to the stars, but all planets occasionally move westward in *retrograde* motion. Mercury and Venus exhibit retrograde motion when they are in conjunction. Mars, Jupiter, and Saturn exhibit retrograde motion when they are in opposition.
- All planets appear brightest when they are in retrograde motion.
- We have determined the following orbital periods for all six Copernican planets: Mercury (88 days), Venus (225 days), Earth (365 days), Mars (686 days), Jupiter (4283 days), and Saturn (10613 days).

3 Inferior or Superior?

Go to a computer and run the CopernicanSystem program. From the `Select Planet` menu choose `User Defined`. In the control window that pops up, set the `Orbit Radius` to 0.7 and the `Orbit` ω (which controls the speed of the orbit) to 2. **Make sure to hit Enter after you type in each value**. Then play the simulation and answer the following questions. If you'd like you can adjust the `Time Step` to make the simulation run slower or faster.

- The Orbit Frame shows a view of the orbits of Earth and the planet from "above" Sun. Earth's orbit is shown in blue, the planet's orbit should be in yellow. A "line of sight" arrow shows where the planet would appear against the stars as seen from Earth (recall that the celestial sphere must be VASTLY larger than Earth's orbit). A planet whose orbit is larger than Earth's is called a *superior* planet. A planet whose orbit is smaller than Earth's is called an *inferior* planet. Is this planet inferior or superior?

- The Sky View Frame shows the apparent motion of Sun and planet as seen from Earth. Watch the Sky View Frame for a while. Does the planet exhibit retrograde motion? If so, does this happen when the planet is in opposition, in conjunction, or at some other time?

- Try to watch the Sky View Frame and the Orbit Frame. See if you can understand the relationship between the two different views. Can this planet EVER be in opposition? Or does it have a limited range of elongations? Explain the reasons for your answer.

- What is happening in the Orbit Frame when the planet exhibits retrograde motion in the Sky View Frame?

- Does the planet appear brightest when it is in retrograde motion? Why is it brightest at this time?

- Based on what you know of the five visible planets, which of these planets must be inferior planets in the Copernican system?

- Note that the planet is moving through its orbit faster than Earth (so it has a shorter period). Does this agree with what you know about the periods of the planets you named as inferior planets? Explain.

- Could we make this planet move through its orbit slower than Earth and still reproduce the same effects? Let's try. Pause the simulation, change the `Orbit` ω to 0.5, hit Enter, and then play the simulation. Watch for a while. What, if anything, is different about the apparent motion of this planet (as seen in the Sky View Frame) this time? Is it possible that the planets you named as inferior planets might have orbital periods LONGER than that of Earth?

- Now let's see what happens if we make the planet's orbit larger than Earth's. Pause the simulation, change the `Orbit Radius` to 1.5 (leave the `Orbit` ω at 0.5), hit Enter, and play the simulation again. Watch the simulation for a while. Is this planet inferior or superior?

- Does this planet exhibit retrograde motion? If so does this occur when the planet is in conjunction, in opposition, or at some other time?

- Can this planet EVER be in opposition, or does it have a limited range of elongations? Explain the reasons for your answer.

- What is happening in the Orbit Frame when the planet exhibits retrograde motion in the Sky View Frame?

- Does the planet appear brightest when it is in retrograde motion? Why?

- Which of the five visible planets are superior planets?

- Note that the planet is moving through its orbit slower than Earth. Does this agree with what you know about the periods of the planets you named as superior planets? Explain.

- Now change the `Orbit` ω to 1.5 and run the simulation. What is different this time? Is it possible that any of the planets you named as superior planets could have orbital periods SHORTER than Earth's?

4 Measuring the Universe

Now we know which planets have orbits smaller than Earth's (Mercury and Venus) and which planets have orbits larger than Earth's (Mars, Jupiter, and Saturn). But how do we compare the sizes of these orbits to each other or, better yet, measure just how big the orbits are compared to Earth's? Let's do that now!

- You found that inferior planets always stay close to the Sun. But how can we tell if Venus is inside Mercury's orbit, or vice versa? Change the `Orbit Radius` back to 0.7 and the `Orbit` ω back to 2. Watch the simulation again, paying close attention to the planet's maximum elongation. Now change the `Orbit Radius` to 0.4 and play the simulation. What do you notice?

  1. When the planet has a smaller orbit its maximum elongation is greater.
  2. When the planet has a smaller orbit its maximum elongation is smaller.
  3. The size of the planet's orbit has no effect on its maximum elongation.

- Based on this information which of these two inferior planets, Mercury or Venus, has the smaller orbit?

- With a bit of trigonometry we can determine the size of each inferior planet's orbit compared to the size of Earth's orbit. Select `Mercury` from the `Select Planet` menu. Play the simulation until Mercury reaches maximum (eastern or western) elongation. (Note: pause the simulation a bit before Mercury is at maximum elongation and then use the Step button to advance one step at a time.) You should find that the lines connecting Earth, Sun, and Mercury form a right triangle. Sketch the arrangement below (you don't need to draw the orbit circles, just the triangle). Label the line connecting Sun and Earth with the symbol $R_{♁}$. Label the line connecting Sun and Mercury with the symbol $R_{☿}$. Indicate that the angle at Earth's location is 28° (since this is the maximum elongation of Mercury as seen from Earth - make sure you understand why it is *this* angle that is 28°).

- Write down an equation (using a trigonometric function) that relates the 28° angle to the lengths $R_{♁}$ and $R_{☿}$. Ask your instructor if you need help with this.

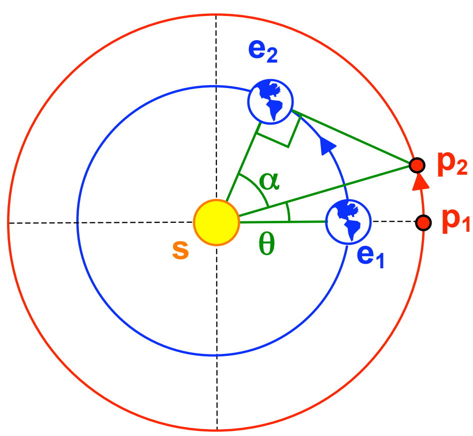
- We know $R_{♁} = 1$ AU (because an Astronomical Unit is just the average distance from Earth to Sun). Solve the equation you wrote down above to determine $R_{☿}$. Record your answer (with units) below.

- Now determine the radius of Venus' orbit ($R_{\venus}$) in AU and record your result below. You can use the `Select Planet` menu to set up the orbit for Venus, but you don't really need to since the geometry is the same except for the maximum elongation angle.

- Determining the relative distances of the superior planets is more challenging. Select `Mars` from the `Select Planet` menu. Play the simulation and pause it when Mars reaches opposition. Sketch the arrangement of Earth, Sun, and Mars in the space below.

- Now play the simulation until Mars reaches quadrature (eastern or western). Recall that quadrature means that planet appears to be 90° away from the Sun in the sky as seen from Earth. So the angle at the location of Earth should be 90°. Sketch this arrangment below.

- If we put the two pictures together we should get something like this ...



In the figure above, label the distance from Earth to Sun $R_{\earth}$ and the distance from Mars to Sun $R_{\mars}$. In the space below, write an equation (using a trigonometric function) that relates $R_{\earth}$ and $R_{\mars}$ to the angle α.

- Now it turns out that we can't measure α directly form observations. But we can still figure it out. First we can find the angle θ. By observation we can determine that it takes 106 days for Mars to move from opposition to quadrature. Through what fraction of a full orbit has Mars moved in this time? Recall that the orbital period for Mars (the time it takes for Mars to go all the way around its orbit) is 686 days.

- Since 360° is a complete orbit, how many degrees has Mars moved through in 106 days? In other words, what is the angle θ in degrees?

$\theta =$ ____

- Now note that the angle $\theta + \alpha$ is just the angle through which Earth has moved in this same 106 days. Determine the value of $\theta + \alpha$ in degrees and record the result below.

$\theta + \alpha =$ ____

- Now we can find α by subtracting the θ from $\alpha + \theta$. Determine alpha and then use your equation from above to find $R_{♂}$. Let $R_{♁} = 1$ AU. Show your work below.

$\alpha =$ ____

- Now you can follow the same procedure to determine the sizes of the orbits of Jupiter and Saturn. First determine the angles θ, $\theta + \alpha$, and α (defined as above) for these two planets. The time between opposition and quadrature (t_Q) and the orbital period (T) is given for each planet in the table below. Complete the table using the procedure you used above for Mars.

Planet	t_Q	T	θ	$\theta + \alpha$	α
Jupiter	87.5 days	4283 days			
Saturn	86.9 days	10613 days			

- Use the same equation you used for Mars and the appropriate value of α from the table above to find the distance from Sun to Jupiter ($R_{♃}$) in AU. Show your work below.

- Now find the distance from Sun to Saturn ($R_{♄}$) in AU. Show your work below.

- Let's put all of your results together. Record the orbital periods you determined in the last activity and the orbital radii you found in this activity in the table below. Give the radius of each planet's orbit in AU.

Planet	Orbital Period (days)	Orbital Radius (AU)
Mercury ☿		
Venus ♀		
Earth ♁		
Mars ♂		
Jupiter ♃		
Saturn ♄		

- What do you notice about the relationship between the orbital radius of a planet and that planet's orbital period?

5 Things to Think About

Copernicus described this relationship as "a wonderful commensurability" indicating "that there is a sure bond of harmony for the movement and magnitude of the orbital circles such as cannot be found in any other way." He was very impressed by this result and it was one his main reasons for believing in the truth of his system. He was also impressed by the fact that his system could be used to determine the relative sizes of the orbits from observational data, saying that his system "binds together so closely the order and magnitudes of all the planets and of their spheres or orbital circles and the heavens themselves that nothing can be shifted around in any part of them without disrupting the remaining parts and the universe as a whole." Contrast this with Ptolemy's system in which the relative size of the epicycle and deferent for a single planet could be found from observation, but there was no way to compare the orbits of two different planets. So Ptolemy could not even be sure about the ordering of the planets! As Copernicus says, Ptolemy was "in exactly the same fix as someone taking from different places hands, feet, head, and the other limbs - shaped very beautifully but not with reference to one body and without correspondence to one another - so that such parts made up a monster rather than a man." This striking description of the Ptolemaic system conjures up images, to the modern mind, of the monster from Mary Shelley's *Frankenstein*. And it was exactly the mathematical beauty of the Copernican system, as compared to the Ptolemaic monstrosity, that would motivate the work of a young man named Johannes Kepler more than 50 years after the death of Nicholas Copernicus.