An Active Introduction to Entropy and Irreversibility for Liberal Arts Students

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ABSTRACT: This poster describes an approach to teaching liberal arts students about the concepts of entropy and irreversibility. The approach uses a hands-on activity involving coins and dice to illustrate the distinction between microstates and macrostates and introduce students to the concept of multiplicity. Entropy is then defined in terms of multiplicity. While completing the activity students will not only see that the model system approaches an equilibrium state, but they will also understand why it does so. This activity is followed by another in which students use open-source computer simulations (developed by the author) to connect what they have learned about entropy and irreversibility to the behavior of ideal gases. These computer simulations also allow students to explore some of the objections that were initially raised against Boltzmann's H-theorem.

Goals of the Activity

- Introduce students to the concepts of microstate, macrostate, and multiplicity.
- Introduce students to Boltzmann's statistical definition of entropy.
- Illustrate, using a simple model, why a system tends to approach equilibrium and why this results in an increase in entropy (in accordance with the Second Law of Thermodynamics).
- Illustrate that fluctuations away from equilibrium become less noticeable as the size of the system is increased.
- Connect these concepts to the behavior of gases: expansion of a gas in a box, mixing of hot and cold gases.
- Allow students to explore historical objections (Maxwell's Demon, Loschmidt's *Umkehreinwand*) and see how these objections are resolved by the statistical approach.

Model System: A Row of Coins

- Consider a row of coins with each coin fixed in place. Each coin can display either heads or tails.
- A *microstate* specifies the exact state of the system in full detail. For our model system this means stating for each coin whether it shows heads or tails.
- A *macrostate* gives a coarse-grained description of the state of the system. For our model system this could mean stating the number of heads and tails.
- The *multiplicity* of a macrostate is the number of microstates that correspond to that macrostate.
- The *probability* of choosing a microstate in a particular macrostate, if the microstate is chosen at random, is just the multiplicity of the macrostate divided by the total number of microstates.
- The *entropy* of a macrostate is proportional to the natural logarithm of the multiplicity: $S = k_B \ln \Omega$ (we can use dimensionless units with $k_B = 1$ for simplicity).
- For a given number of coins students can write out all of the microstates and determine the multiplicity, probability, and entropy for each macrostate.
- For example, the 8 microstates for a row of 3 coins are: HHH, THH, HTH, HHT, HTT, THT, TTH, and TTT. This gives the results shown in the table below:

Macrostate	Multiplicity (Ω)	Entropy ($S = \ln \Omega$)	Probability ($P = \Omega/\Omega(\text{all})$)
3H, 0T	1	0	1/8
2H, 1T	3	1.0986	3/8
1H, 2T	3	1.0986	3/8
0H, 3T	1	0	1/8

- Students can find multiplicities for rows of 1, 2, 3, and 4 coins. These multiplicities can be arranged to form Pascal's Triangle and then students can use the rule for constructing Pascal's Triangle to find multiplicities for larger numbers of coins:

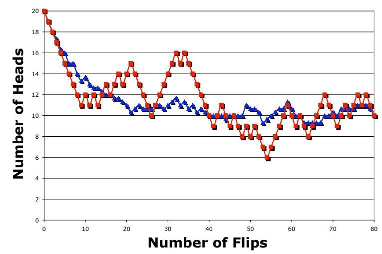
n											
1					1		1				
2				1		2		1			
3			1		3		3		1		
4		1		4		6		4		1	
5	1		5		10		10		5		1

- By examining these multiplicities students can see that macrostates near equilibrium (equal numbers of heads and tails) always have the greatest multiplicities and thus the greatest probabilities and entropies.
- In addition, students can see that as the number of coins increases the probability becomes increasingly concentrated in a ever-smaller fraction of macrostates near equilibrium.

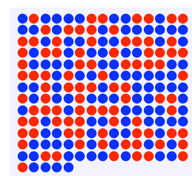
Flipping Coins

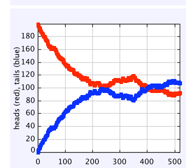
- What happens if we start with a row of 20 coins and begin flipping coins over at random? Students can explore the resulting behavior themselves using a row of 20 pennies and a 20-sided die:
- Students roll the die and flip over the corresponding coin, keeping track of the number of heads and tails after each roll.

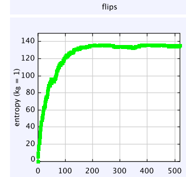
- The red squares in the graph below show the results from 80 rolls by a single group of students. The blue triangles show the results obtained by averaging data from six different groups.



- It is clear that the system gradually approaches equilibrium, but continues to fluctuate about the equilibrium state. The relative size of the fluctuations decreases as the number of coins increases.
- To explore the behavior with larger numbers of coins students use a computer simulation. The simulation provides a graphic representation of the system of coins (red = heads, blue = tails) as well as graphs of the number of heads/tails and the entropy.
- The figures on the right show a snapshot from the simulation with 200 coins.
- Using the simulation students can show that in all cases the system tends to approach equilibrium and that, as a result, the entropy of the system generally increases.
- As the number of coins is increased the occasional fluctuations away from equilibrium (and the corresponding decreases in entropy) decrease in relative size.
- For very large numbers of coins the fluctuations are no longer noticeable and the entropy appears to increase steadily until the system reaches the equilibrium state, at which point the entropy levels off and remains constant.
- So in this model system the Second Law does not hold absolutely, but the probability that it holds approaches 1 as the system increases in size.

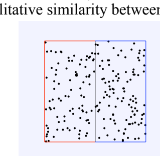


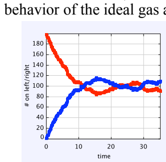




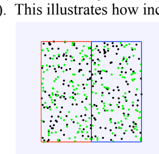
From Coins to Ideal Gases

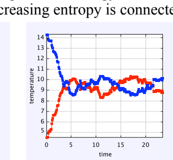
- Students can then explore simulations of an ideal gas in a box to see how the behavior of the coin model relates to the behavior of a more realistic model system.
- The first simulation starts with an ideal gas confined to the left side of a box. The gas expands, eventually reaching equilibrium (and maximum entropy) when it fills the box.
- The simulation shows a graphical representation of the gas, as well as a graph showing the number of molecules on each side of the box (as shown below, for 200 molecules). Students can readily see the qualitative similarity between the behavior of the ideal gas and that of the coin model.



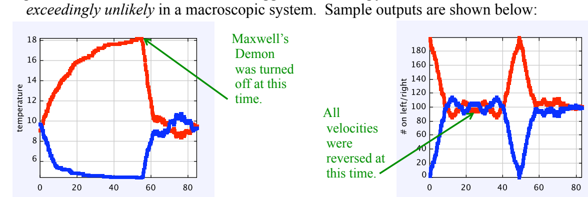


- Another simulation (sample snapshot shown below) allows students to observe the behavior of a system that starts with a hot gas on one side of the box and a cold gas on the other. The gases expand and overlap (without interacting), leading both sides of the box to reach an intermediate temperature (as measured by the average kinetic energy of the molecules on each side of the box). This illustrates how increasing entropy is connected with heating.





- The simulations can also be used to illustrate some historical objections to Boltzmann's ideas such as Maxwell's Demon (which sorts fast and slow molecules and thus reverses heating) and Loschmidt's *umkehreinwand* (in which a time-reversed gas shows a decreasing entropy). The simulations illustrate that violations of the Second Law are *possible*, but Boltzmann's statistical approach to entropy assures us that such violations are *exceedingly unlikely* in a macroscopic system. Sample outputs are shown below:



- By completing this activity students can gain insight into the statistical interpretation of entropy and understand why Boltzmann claimed that the Second Law "means nothing else than that … the system of bodies goes from a more improbable to a more probable state." (W. F. Magie, *A Source Book of Physics*, p. 263).

Resources

- Handouts for the activity and all computer simulations can be downloaded for free from `http://facultyweb.berry.edu/ttimberlake/entropy`.
- 20-sided dice can be purchased at many hobby stores.
- For more detailed information please see my paper describing this activity (available for download at `http://facultyweb.berry.edu/ttimberlake/entropy`) and the references therein.