Correlation of Potodetachment Rate and Lyapunov Exponent for a Scarred Resonance State

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Overview

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III. Quantum Calculations

IV. Husimi Distributions

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Model and Previous Results

Driven Inverted Gaussian Hamiltonian (Radiation Gauge)

$$H = \frac{1}{2}\left(p - \frac{\epsilon}{\omega}\sin(\omega t)\right)^2 - V_0 \exp(-(x/a)^2) \tag{1}$$

$V_0 = 0.63$ a.u. and $a = 2.65$ a.u.

ω is the frequency and ϵ is the strength of the driving field

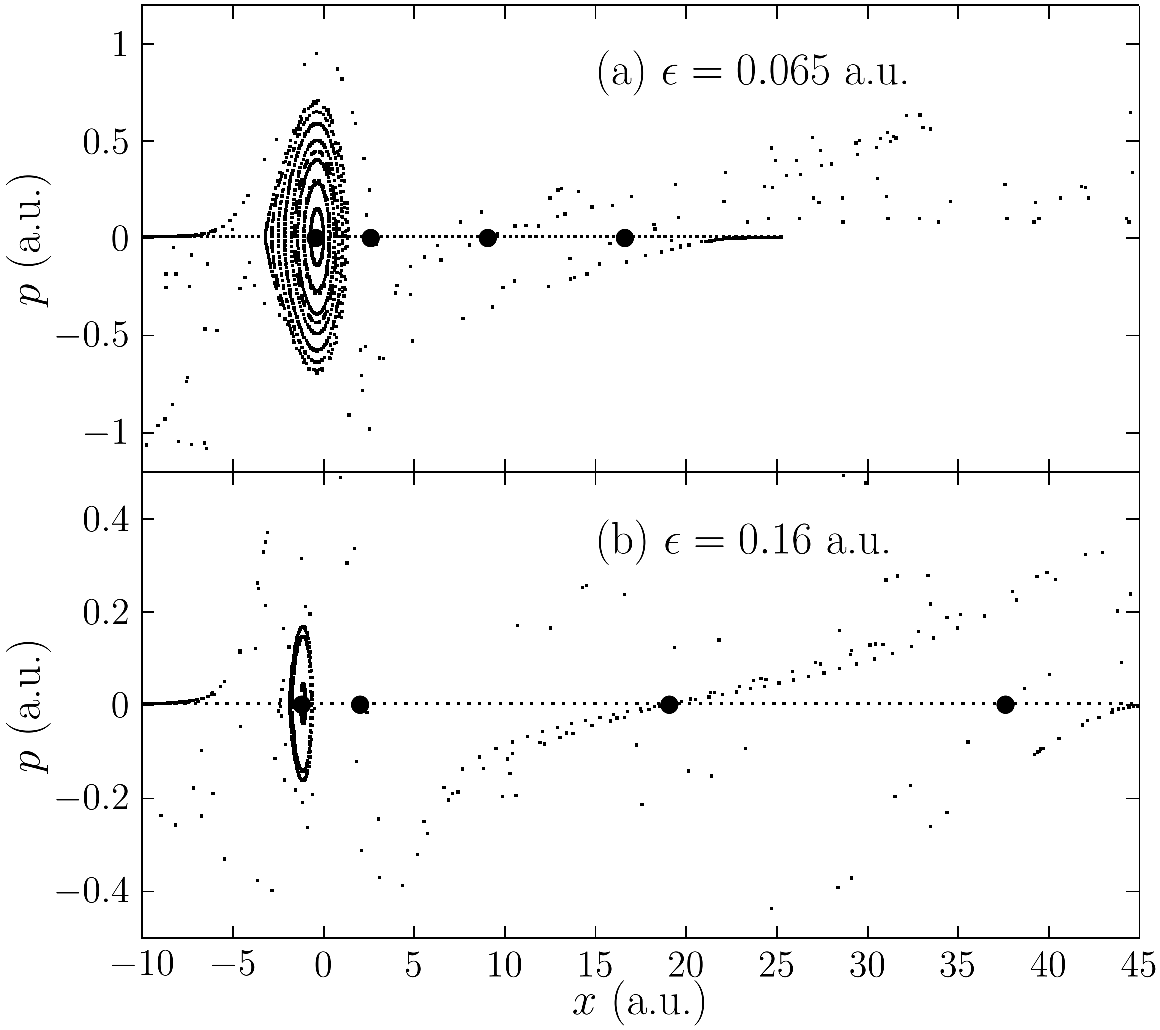
$\omega = 0.0925$ a.u., ϵ is varied

Previous Results

N. Ben-Tal, N. Moiseyev, and R. Kosloff, found that the number of resonance states in the system increases as ϵ is increased (J. Chem. Phys. **98**, 9610 (1993)) → atomic stabilization

I found that the newly created resonance states are scarred on unstable periodic orbits of the classical motion (PRA **64** 033404 (2001)) → connection between scarring and stabilization

Classical Dynamics

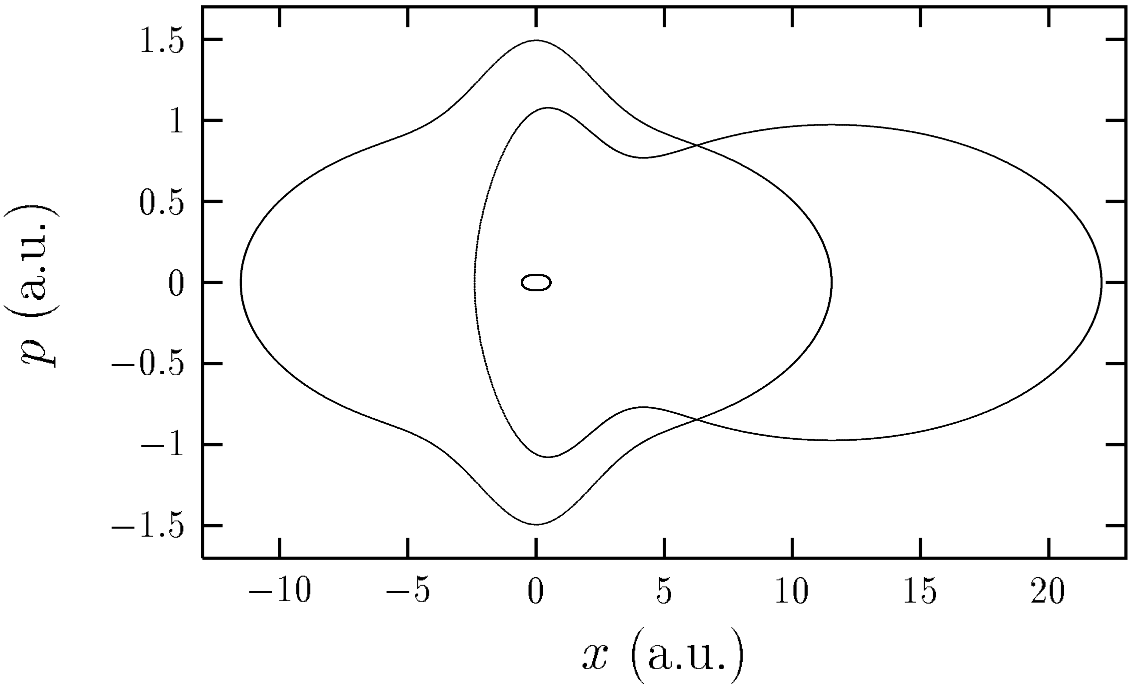


Periodic Orbits (filled circles, A to D from left to right)

One stable periodic orbit (Orbit A) surrounded by a region of regular motion that diminishes in size as ϵ is increased.

Three unstable periodic orbits (B through D) move apart along $p = 0$ as ϵ is increased.

Overall Motion: Increasingly chaotic as ϵ is increased.

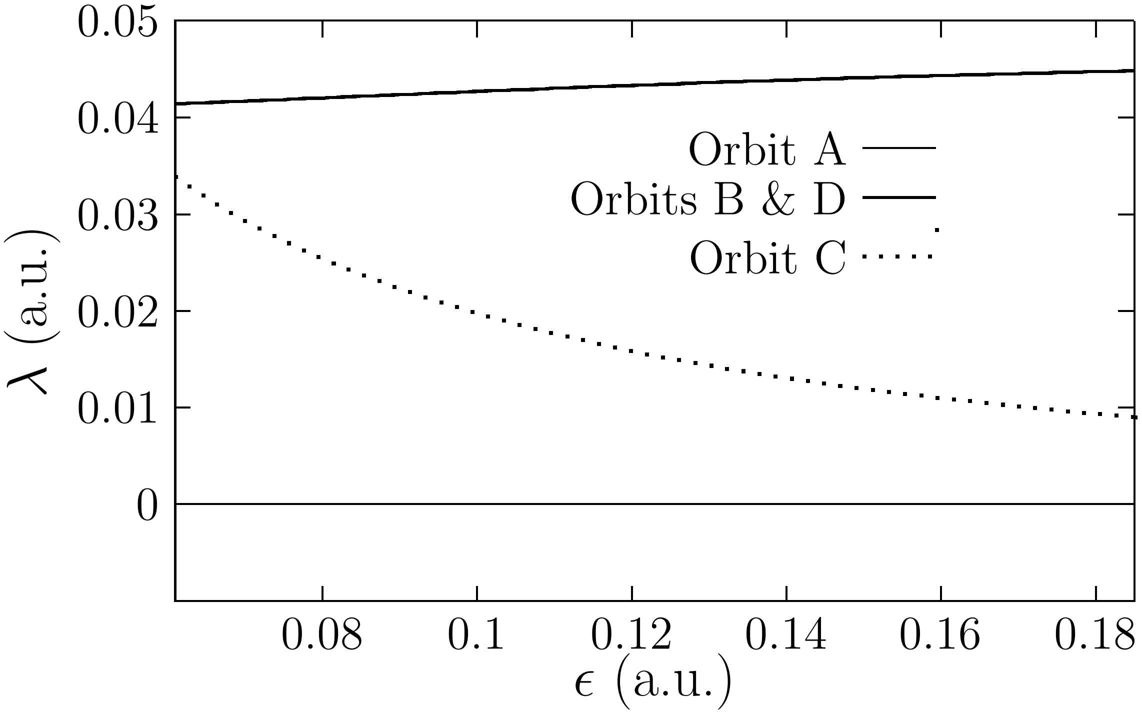


Periodic Trajectories

Stable trajectory (Orbit A) is the small oval near the origin.

Orbits B (not shown) and D are mirror images about $x = 0$.

Orbit C is symmetric about $x = 0$.



Lyapunov Exponents (λ)

Orbit A is stable for 0.065 a.u. $< \epsilon <$ 0.18 a.u.

Orbits B and D become less stable as ϵ is increased.

Orbit C becomes more stable as ϵ is increased.

Quantum Calculations

The quantum system is characterized by eigenstates of one-period time evolution operator → Floquet states

We are interested in metastable resonance states that interact strongly with the potential well.

System can ionize → must use special techniques.

Regular complex scaling: $x \rightarrow xe^{i\theta}$

Gives complex quasienergies: $\Omega + i\Gamma/2$, where lifetime is $\tau = 1/\Gamma$.

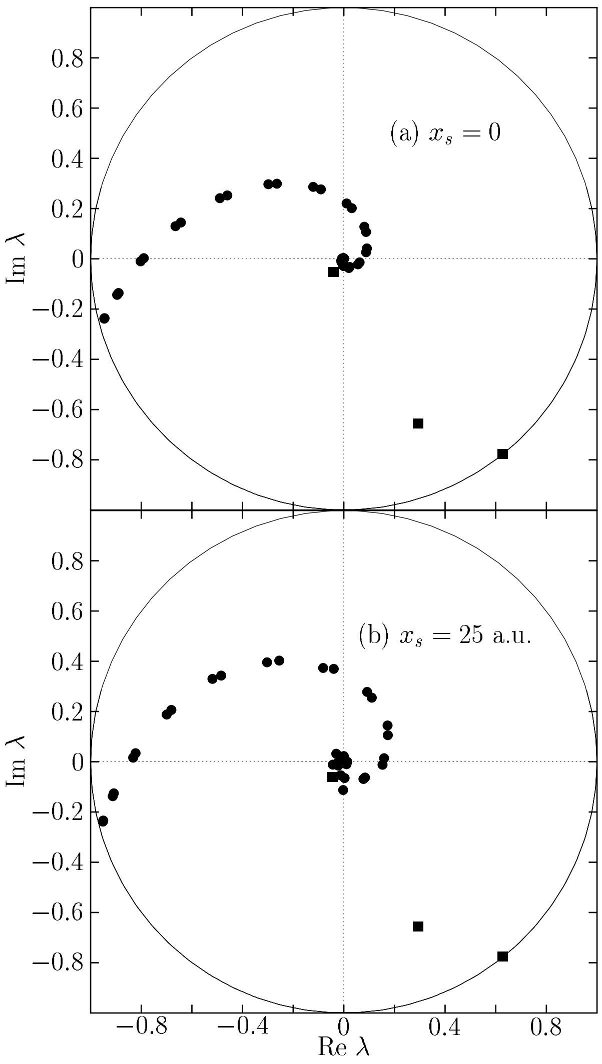
Used to identify resonances and determine photodetachment rates (Γ).

Distorts wavefunctions.

Exterior complex scaling: coordinate scaled only for $|x| \geq x_s$

$x_s = 25$ a.u. for low field strengths, $x_s = 50$ a.u. for high field strengths

Used to calculate correct wavefunctions in unscaled region ($|x| < x_s$).



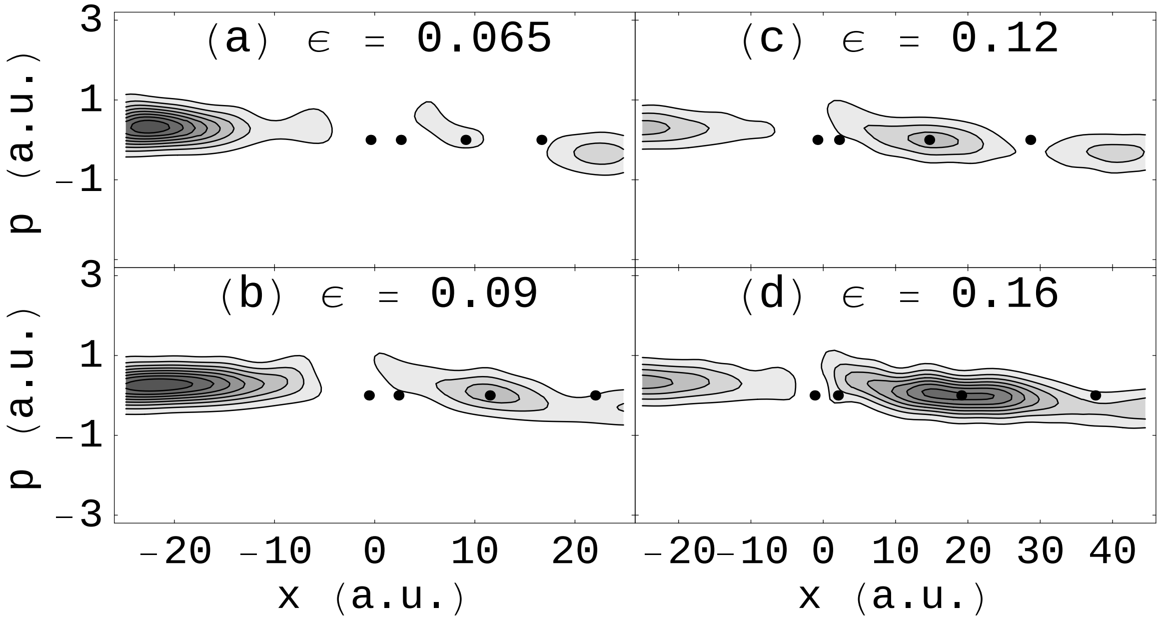
Eigenvalue Plots

Resonance states are off the spiral. Closer to center implies shorter lifetime.

Top → regular scaling. Converges rapidly, easy to identify resonances.

Bottom → exterior scaling. Must use larger basis, spiral not as well-defined, hard to identify resonances. Worse as ϵ is increased.

Husimi Distributions

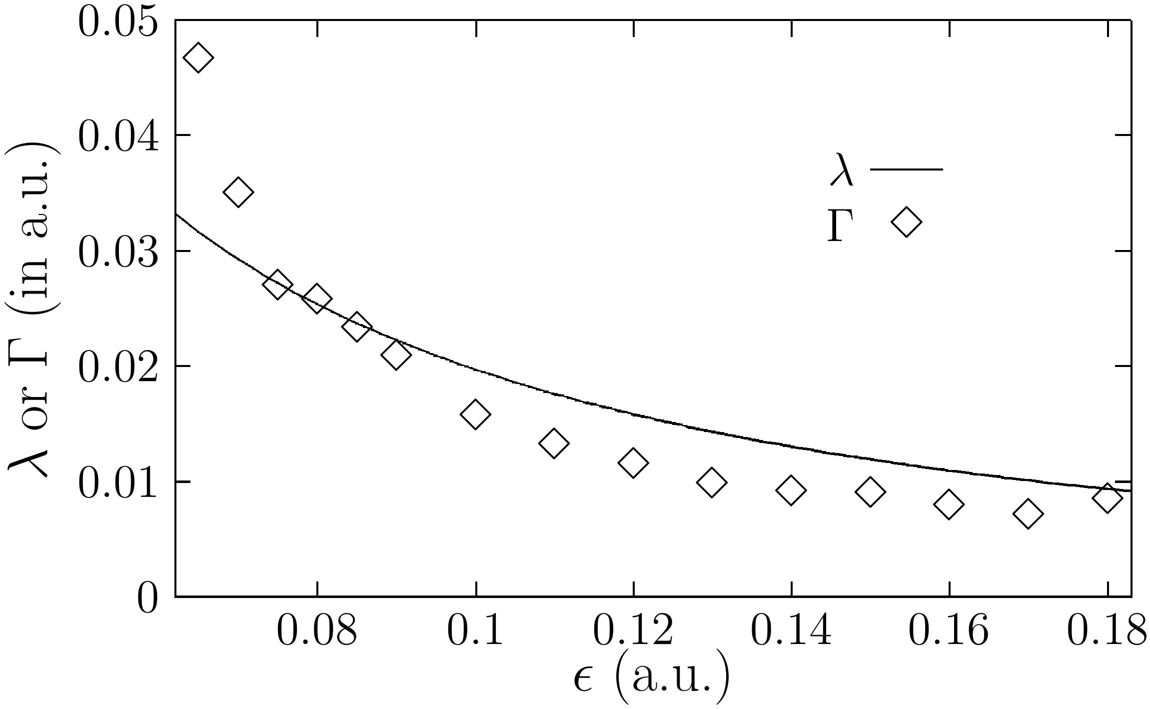


To visualize the phase-space structure of a Floquet state we plot its Husimi Distribution.

The Husimi Distribution is a smoothed probability distribution in phase space. Smoothing meets requirements of Uncertainty Principle.

These Husimi Distributions are for one resonance state that is scarred on the unstable Orbit C. The locations of periodic orbits are shown as filled circles.

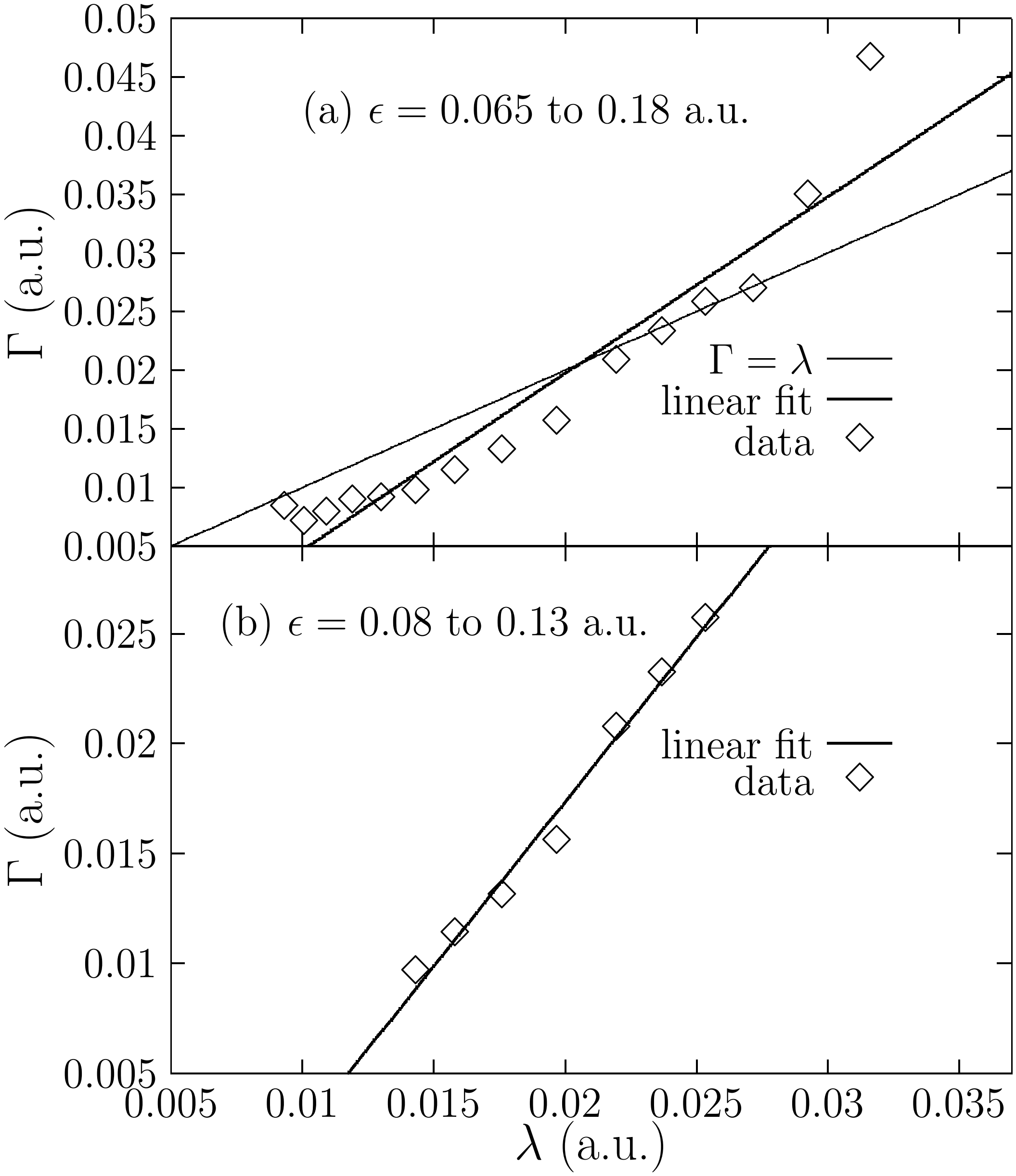
Correlation of Photodetachment Rate and Lyapunov Exponent



Continuous line shows Lyapunov exponent (λ) of Orbit C.

Data points show photodetachment rate (Γ) of scarred resonance state.

Note similar behavior as ϵ is increased.



Correlation Plot

Strong correlation between λ and Γ ($R = 0.953$) over full range.
Best-fit line: $\Gamma = 1.505\lambda - 0.010$ a.u.

Relationship is even more linear ($R = 0.993$) over restricted range.
Best fit for restricted range: $\Gamma = 1.496\lambda - 0.013$ a.u.

Conclusion

New type of quantum-classical correspondence in an open, chaotic system.

Photodetachment rate of a resonance state scarred on a single unstable periodic orbit is strongly correlated with the Lyapunov exponent of that orbit.

Relationship between photodetachment rate and Lyapunov exponent is more linear for a restricted range of field strengths.

We expect the resonance state to be most closely associated with Orbit C in this restricted range: lower $\epsilon \rightarrow$ still forming, higher $\epsilon \rightarrow$ spreading to other orbits.

No strong correlation seen for other resonance states → associated with more than one orbit, including stable orbit.

We would like to see if the photodetachment rate and Lyapunov exponent become identical as $\hbar \rightarrow 0$.