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Eigenvalue Spacings in the Asymmetric Infinite Square Well

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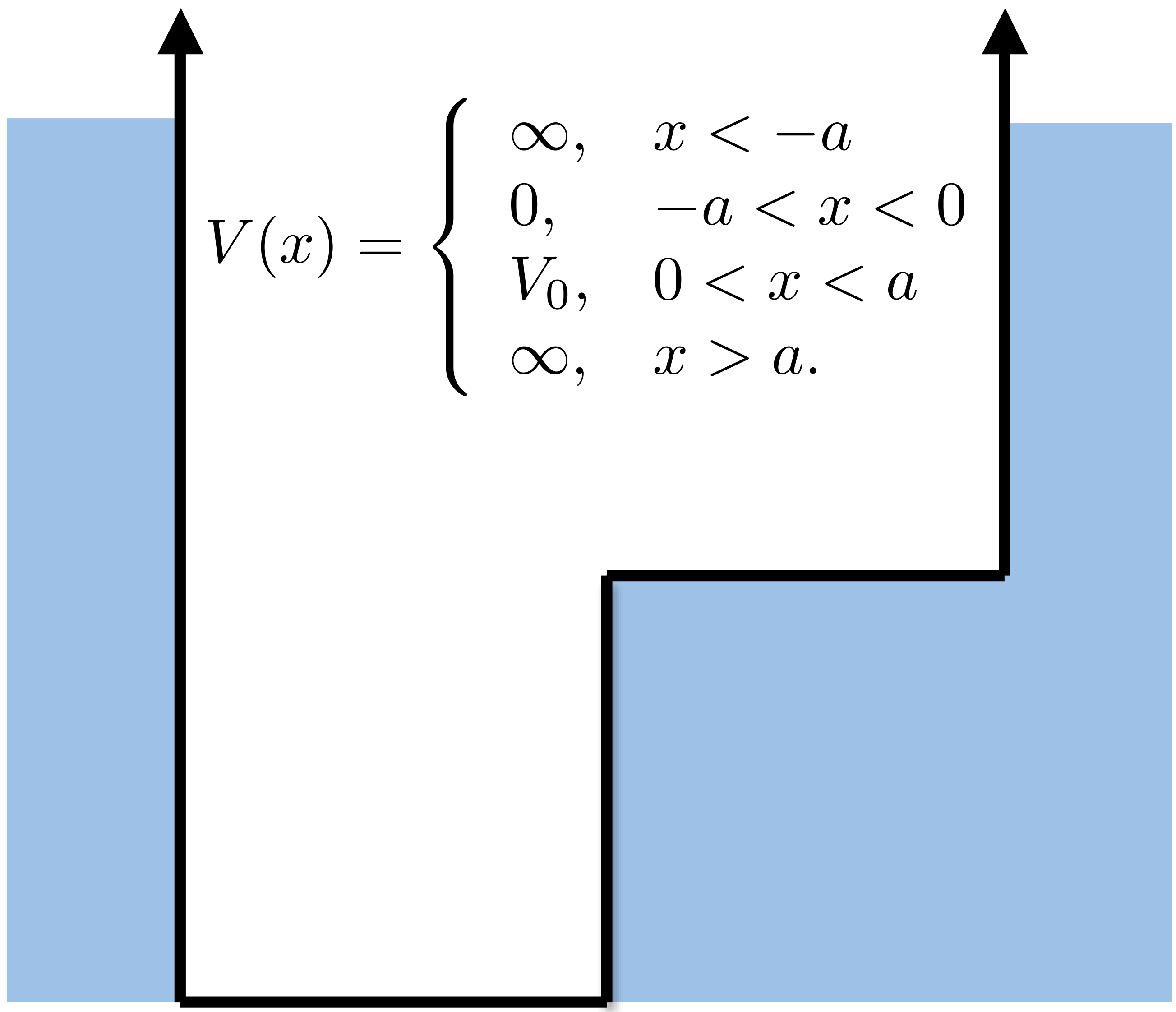
Asymmetric Infinite Square Well (AISW)



$$V(x) = \begin{cases} \infty, & x < -a \\ 0, & -a < x < 0 \\ V_0, & 0 < x < a \\ \infty, & x > a. \end{cases}$$

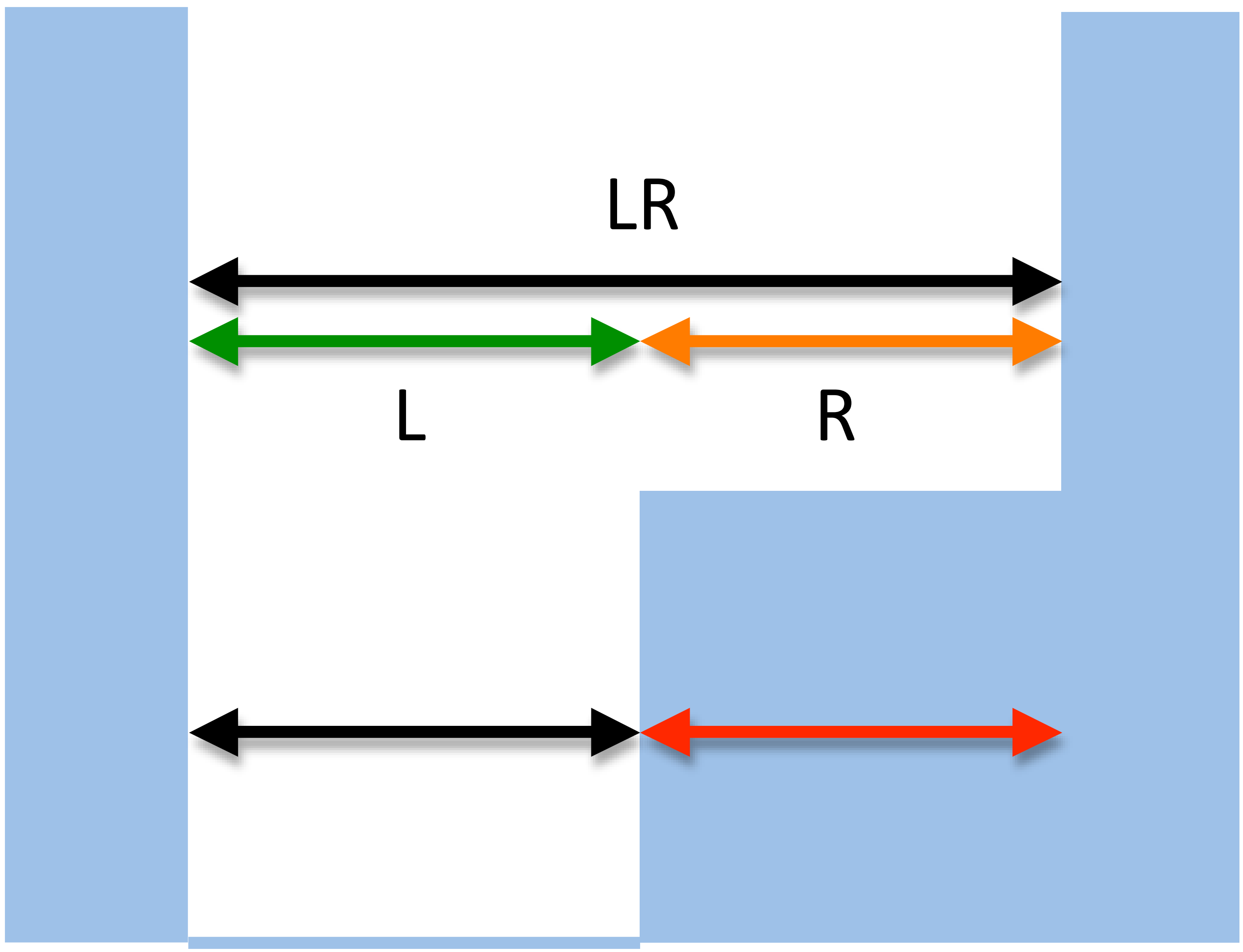
Reflection Coefficient

- Using elementary quantum mechanics we can find the reflection coefficient at the step:

$$r = \frac{1 - \sqrt{1 - V_0/E}}{1 + \sqrt{1 - V_0/E}}$$

- Note that $\hbar$ does not appear in this formula, so this result persists in the classical limit $\hbar \longrightarrow 0$.
- So reflection is possible in the classical dynamics even for $E > V_0$.

Non-Newtonian Dynamics



Semiclassical Energy Formula

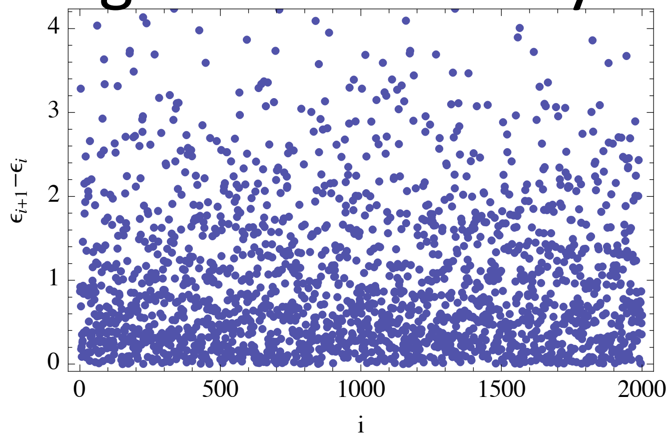
- Semiclassical theory allows us to determine the (unfolded) energy eigenvalues of the quantum system using information about the classical dynamics:

$$\epsilon_n = 2n - 1/2 - \gamma_0 - \frac{1}{\pi}\int_{\pi(n-1/2)}^{\pi(n+1/2)} \left(\frac{S}{\pi} - \gamma_0\right) dS$$
$$- \frac{1}{\pi^2} \mathrm{Im} \sum_{p,\nu} \int_{\pi(n-1/2)}^{\pi(n+1/2)} \frac{A_p^\nu}{\nu} e^{i\nu S_p} dS$$

- Are there features of the quantum spectrum that are connected to the non-Newtonian orbits?

Eigenvalue Spacing Distribution

- Regular classical dynamics: Poisson



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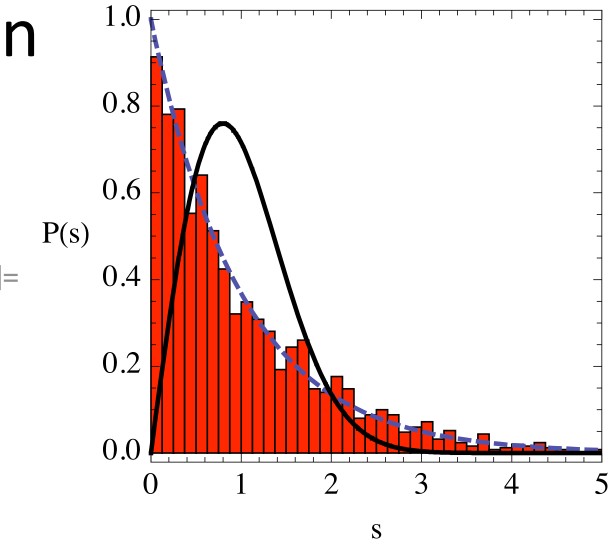
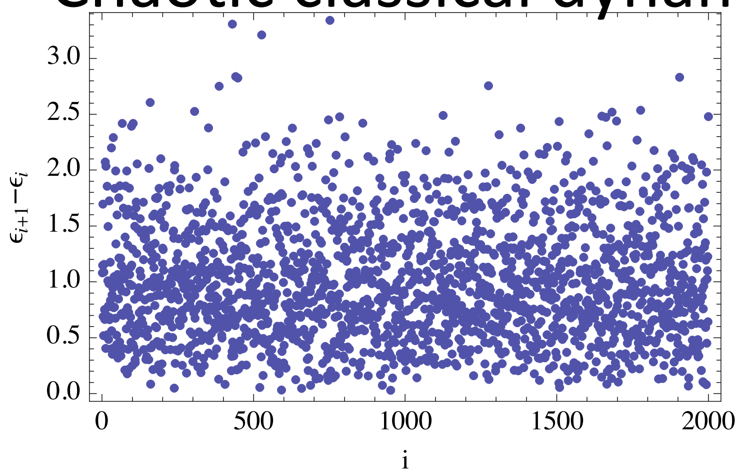
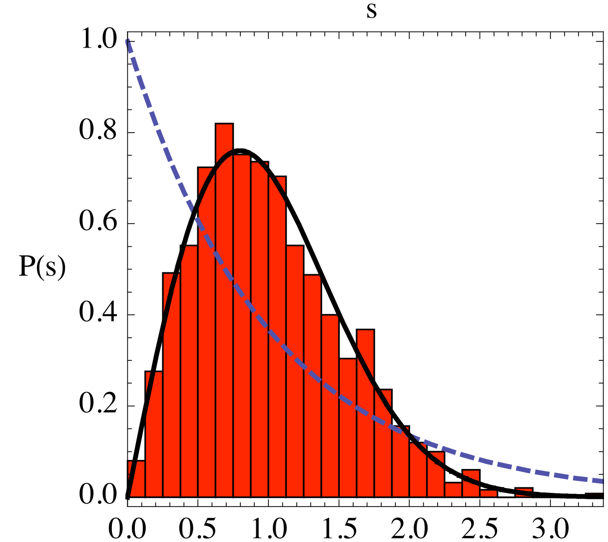


- Chaotic classical dynamics: GOE

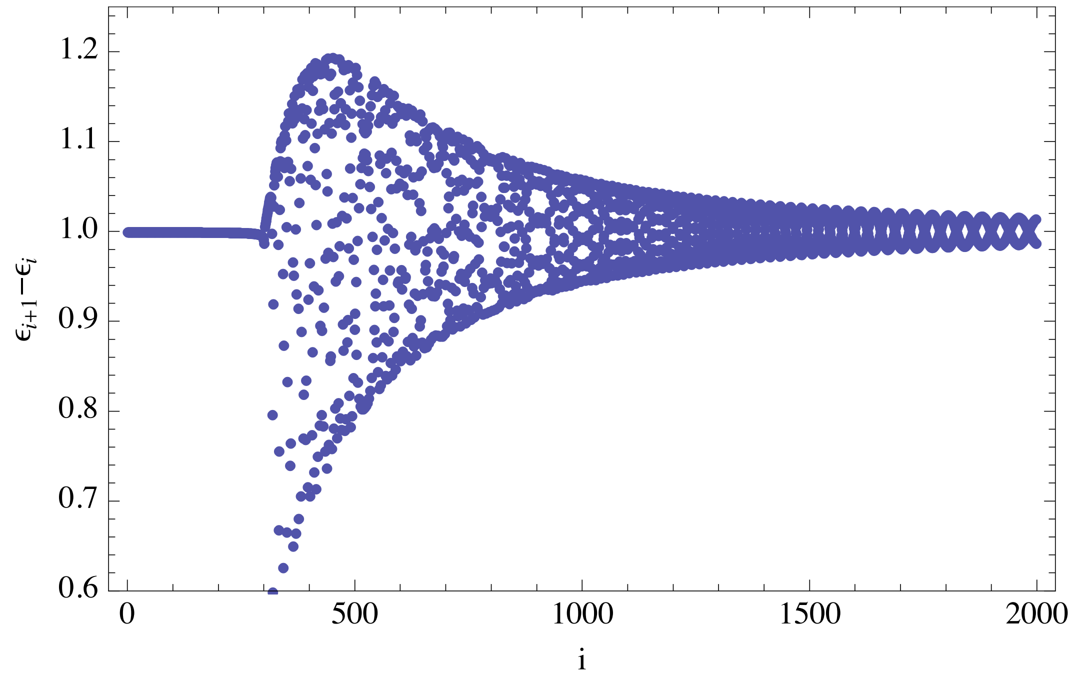


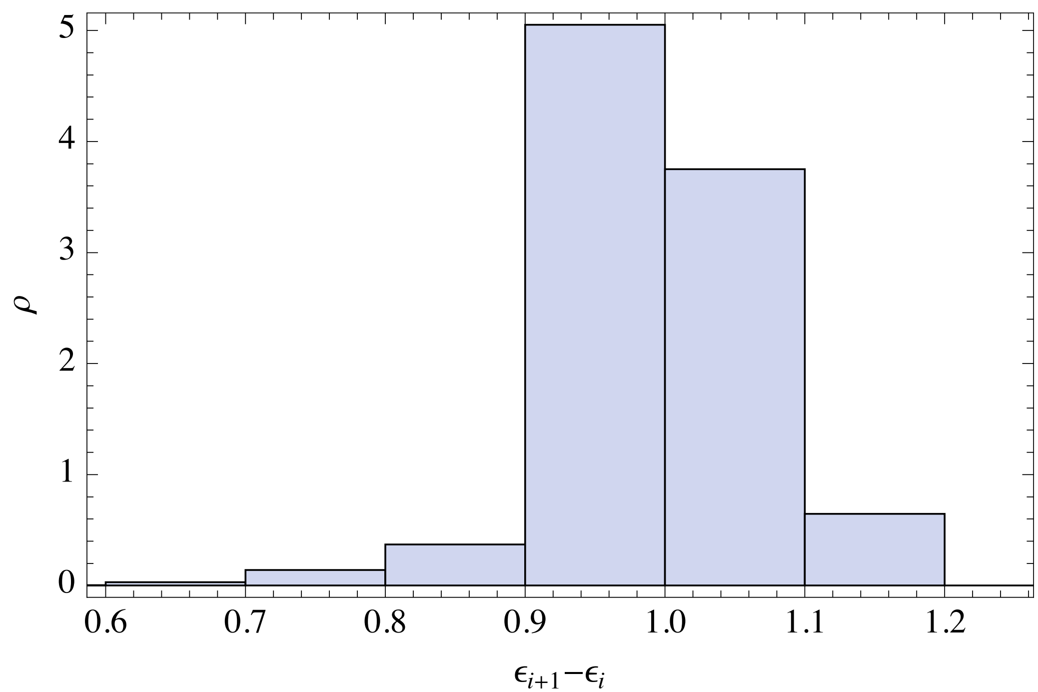


- 1-D systems: uniform spacings (after unfolding)

Computed Spacings

- Numerically solved the transcendental equation for the energy eigenvalues.
- Unfolded the eigenvalues
- Calculated spacings.
- Not uniform!





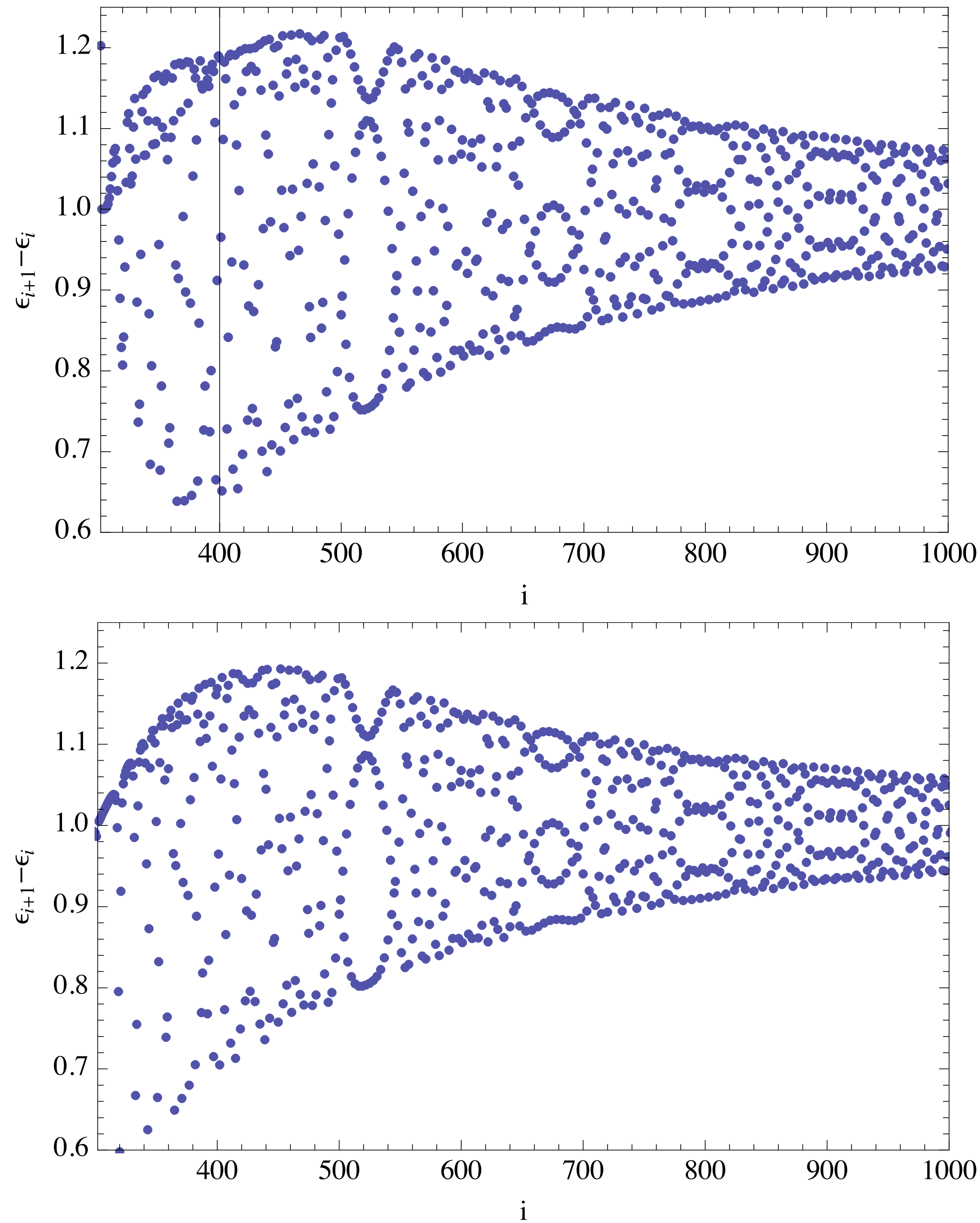
Semiclassical Spacings

- $\epsilon_{n+1} - \epsilon_n = 1 - (\omega_{n+1} - \omega_n)$ where

$$\omega_n = \frac{1}{\pi^2} \mathrm{Im} \sum_{p,\nu} \int_{\pi(n-1/2)}^{\pi(n+1/2)} \frac{A_p^\nu}{\nu} e^{i\nu S_p} dS$$

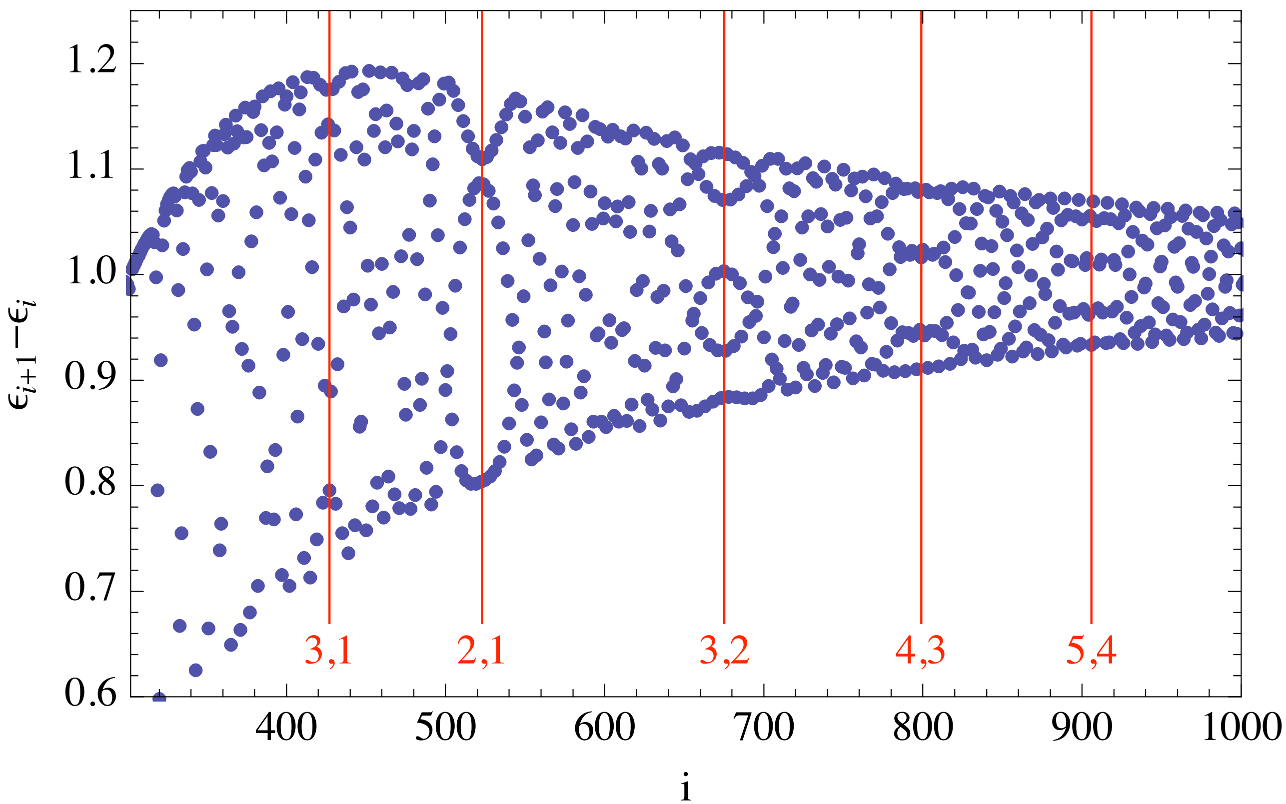
- For Newtonian orbit LR, $\omega_n \approx 0$, so non-uniformity is due to non-Newtonian orbits.
- For $E >> V_0$: $r \approx 0 \rightarrow A_p^\nu \approx 0$ for the non-Newtonian orbits, so spacing is uniform.

- Good agreement between full quantum (bottom) and semiclassical (top) spacings using short non-Newtonian orbits (L, R, LL, RR) for $E > V_0$.
- Note resonance features.



Applying Resonance Criterion

- Can use the semiclassical formula to determine (unfolded) energies at which resonances occur for various values of (k,j).



Conclusion

- AISW exhibits non-uniform spacing (after unfolding) for energies just above step height.
- Deviations from uniformity are due to non-Newtonian classical orbits.
- Semiclassical predictions match computed quantum results, including resonance features.

References

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