Signatures of Chaos in Periodically Driven Quantum Systems

by

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Introduction

The Model

Kramers-Henneberger Potential

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Complex Coordinate Scaling

Floquet Theory

Phase-space structure

Avoided crossing

Conclusion

Introduction

Interest: Quantum versions of classically chaotic systems.

Signatures of "quantum chaos":

Eigenvalue statistics change from Poisson to Random-Matrix.
Eigenvalue spectrum is related to periodic orbits. (Gutzwiller)
Eigenstates can be scarred on periodic orbits. (Heller)
Eigenstates can be exponentially localized without classical barriers. (Raizen, et. al.)
System can be stable against ionization in intense fields: classical (Dunning, et. al.) and non-classical
Our model: inverted gaussian potential with periodic driving field.

The number of metastable resonance states increases as strength of driving field is increased.
The classical dynamics becomes increasingly unstable as driving field is increased.
This is an indication of non-classical stabilization.

The Model

Driven Inverted Gaussian Hamiltonian

$$H = \frac{1}{2}\left(p - \frac{\epsilon}{\omega}\sin(\omega t)\right)^2 - V_0 \exp(-(x/a)^2) \tag{1}$$

$V_0 = 0.63$ a.u. and $a = 2.65$ a.u.

ω is the frequency and ϵ is the strength of the driving field

$\omega = 0.0925$ a.u., ϵ is varied

Prior Studies

This model was investigated by N. Ben-Tal, N. Moiseyev, and R. Kosloff, who found that the number of resonance states in the system increases as ϵ is increased (J. Chem. Phys. **98**, 9610 (1993)).

In this study Ben-Tal, et. al. explained the increased number of resonances by refering to the Kramers-Henneberger time-averaged potential.

Continuum states form a spiral, which moves as the scaling angle (θ) is changed.

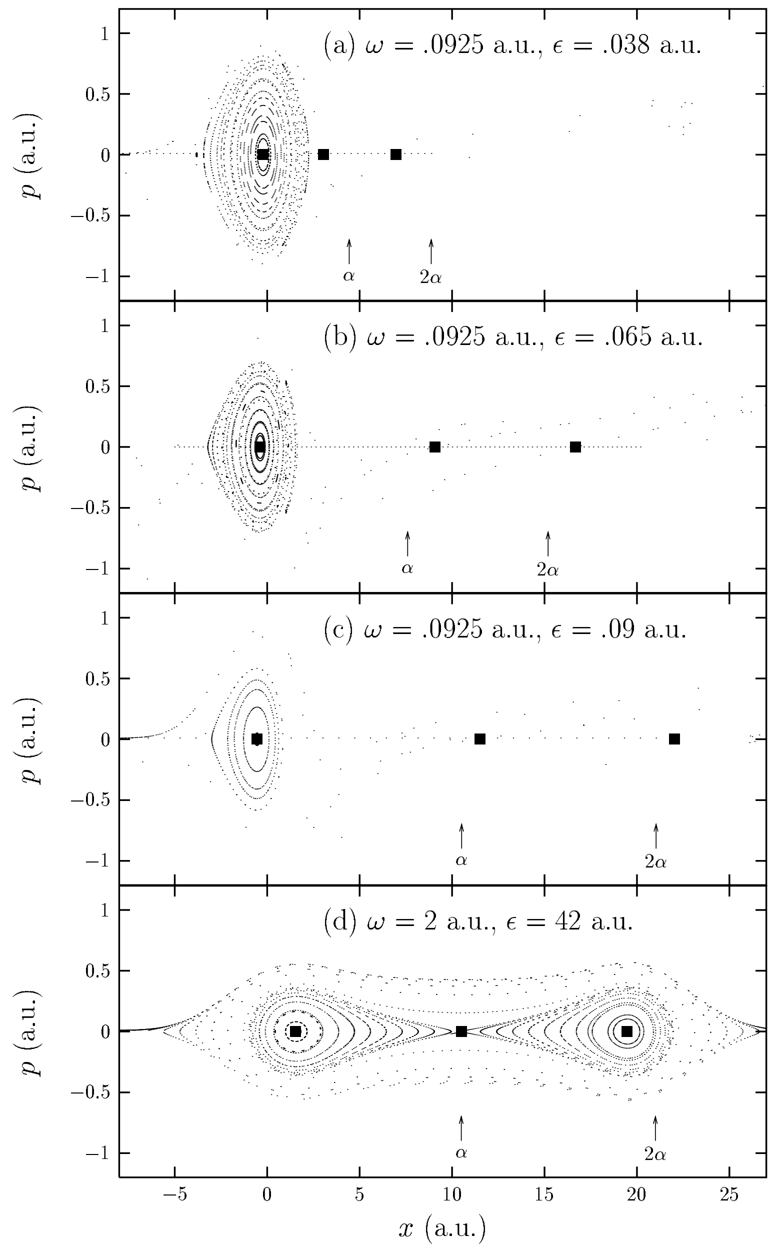
Resonance states are not on the spiral and don't move when θ is changed.

The continuum states are indicated by circles and the three resonance states by squares.

4 resonance states

5 resonance states

Classical Motion

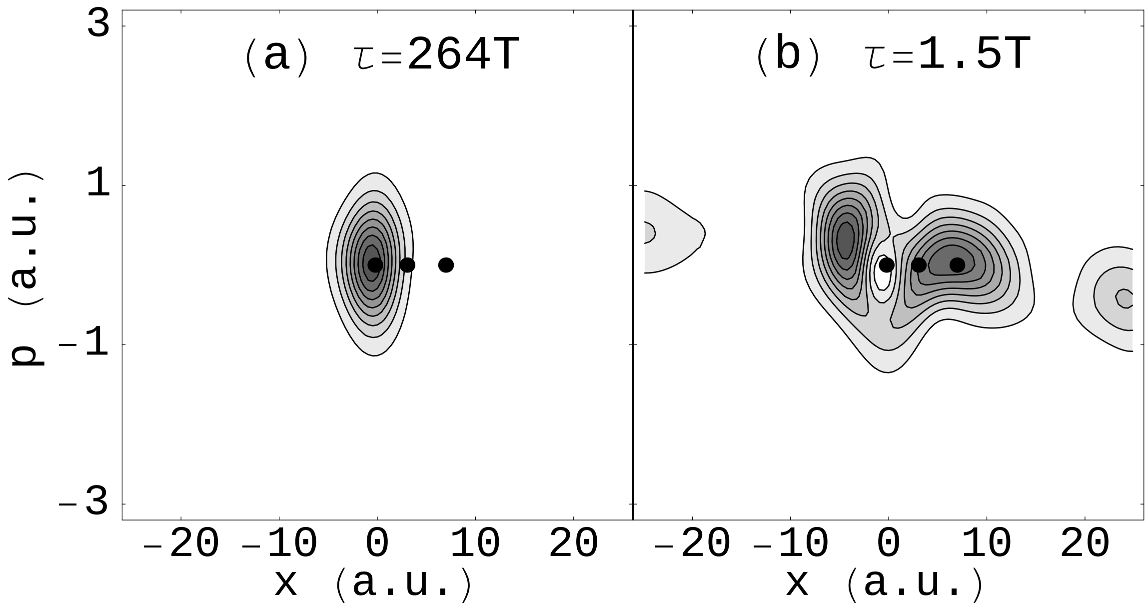


Time-averaged K-H approximation valid for frequencies much larger than the bound state energies (.445, .140, .0001 a.u.).

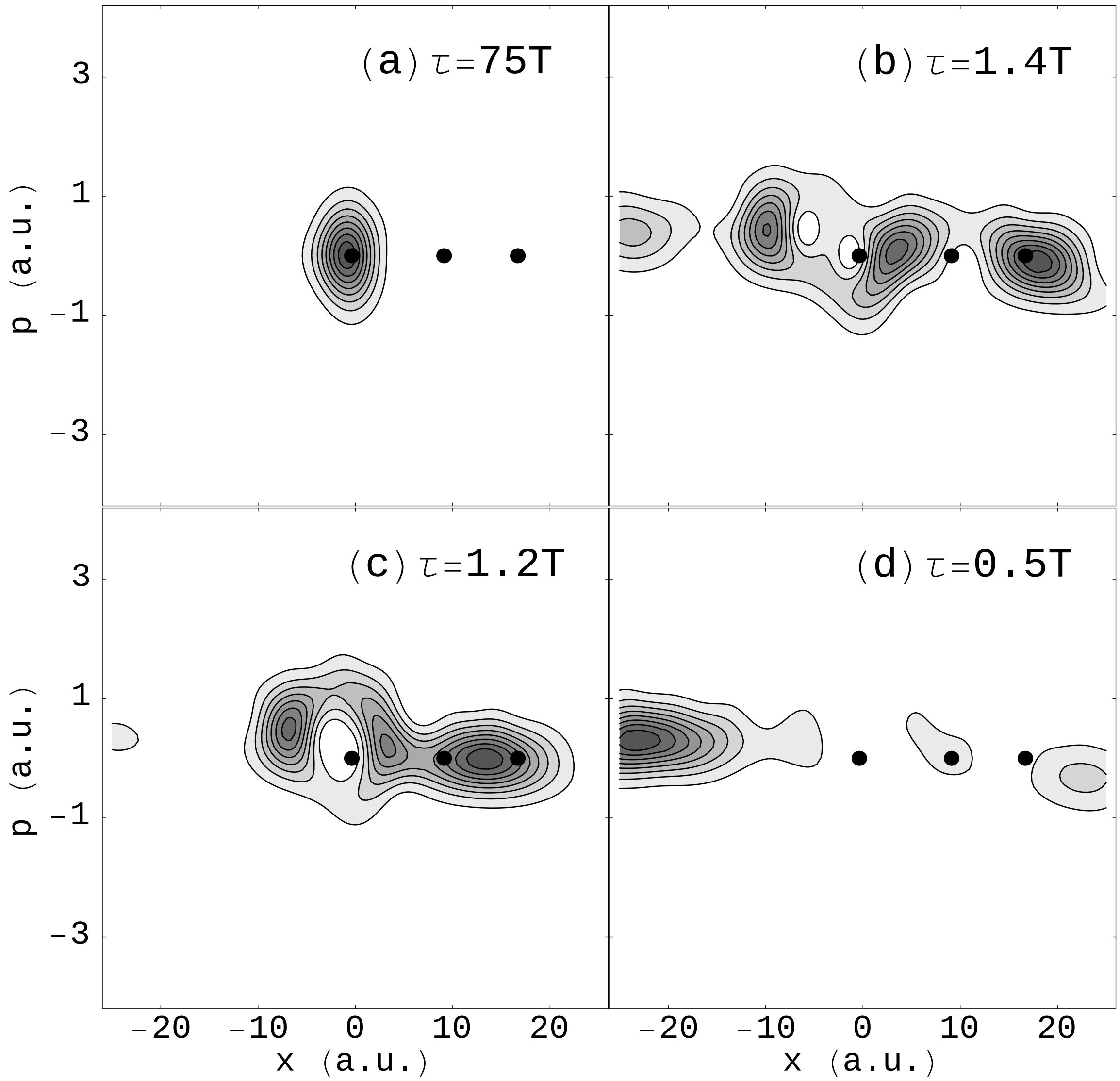
Valid approximation for $\omega = 2$ a.u., but not for $\omega = 0.0925$ a.u.

So why does the number of resonances increase even though the classical motion is becoming less stable?

Husimi Distributions of Resonances

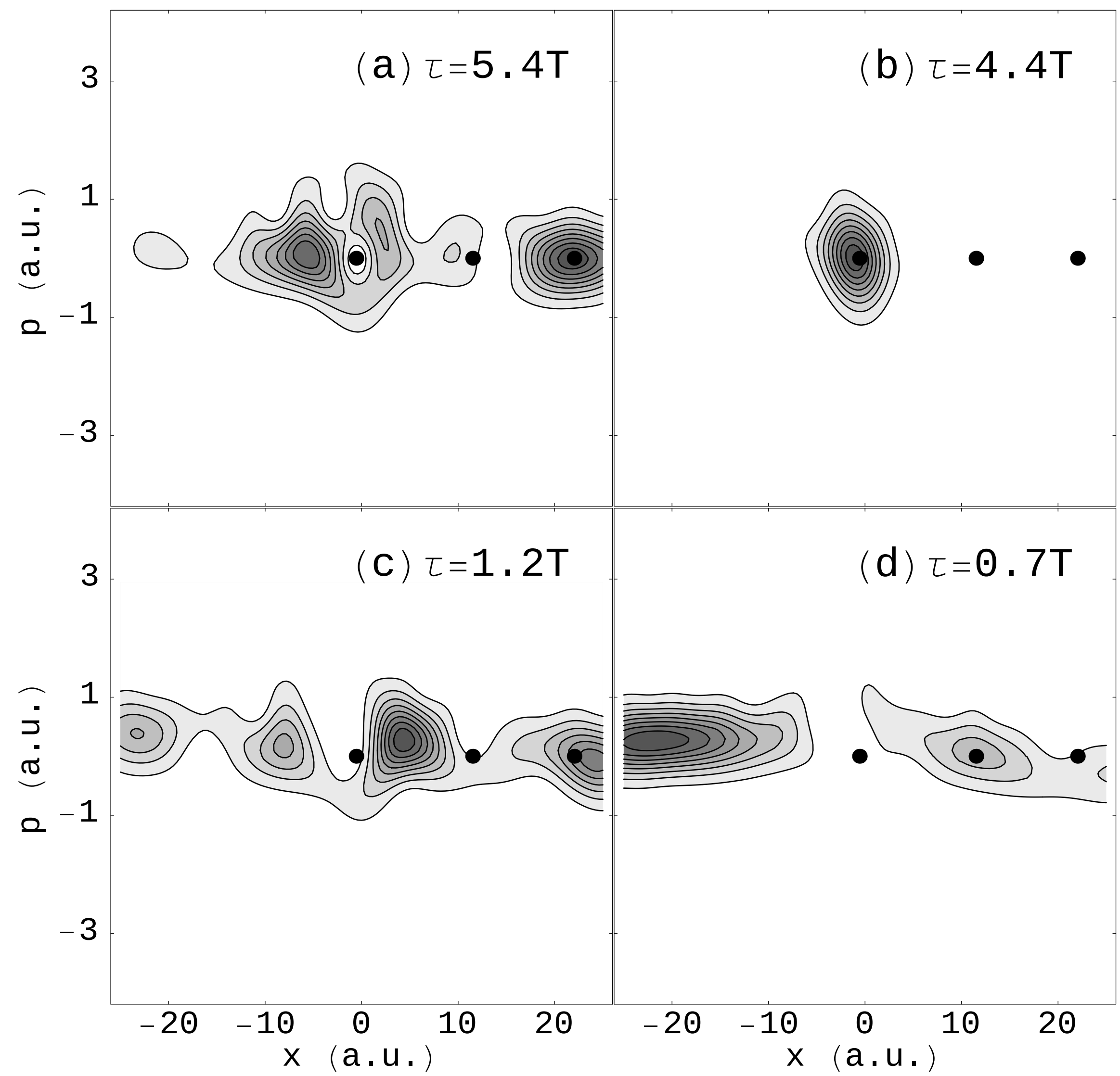


At $\epsilon = 0.038$ a.u. the resonance states are localized inside the stable classical region, but are beginning to stretch toward the unstable periodic orbits.



At $\epsilon = 0.065$ a.u. all but 1 state is significantly stretched toward the unstable periodic orbits.

The newly created resonance state has a modest peak on one of the unstable periodic orbits.



At $\epsilon = 0.09$ a.u. all of the states have peaks on periodic orbits.

The new state has a larger lifetime and a stronger peak on the unstable periodic orbit.

Conclusion

Scarring of resonance states on unstable periodic orbits allows the number of resonance states to increase as ϵ is increased.

At low ϵ all of the periodic orbits are close together in phase space and this allows only a few states to be supported on these periodic orbits.

At high ϵ the periodic orbits are spread out and cover a larger region of phase space, allowing more quantum states to be supported.

This effect might stabilize excited states of this system in intense fields, but it is unlikely to stabilize the ground state.