Quantum-Classical Correspondence in Driven Open Systems

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References: *PRA* **64**, 033404 (2001)
PRL **90**, 103001 (2003)

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Introduction & Model

- Goal: Study quantum-classical correspondence in open, chaotic systems
- Model: Driven Inverted Gaussian

$$H = \frac{1}{2}\left[p - \frac{\varepsilon}{\omega}\sin\omega t\right]^2 - V_0 \exp\left[-(x/a)^2\right]$$

  – In atomic units: $V_0 = 0.63$, a = 2.65, □ = 0.0925
  – Study behavior for various values of □
- Prior to our work this model was known to exhibit behavior similar to atomic stabilization.

Atomic Stabilization

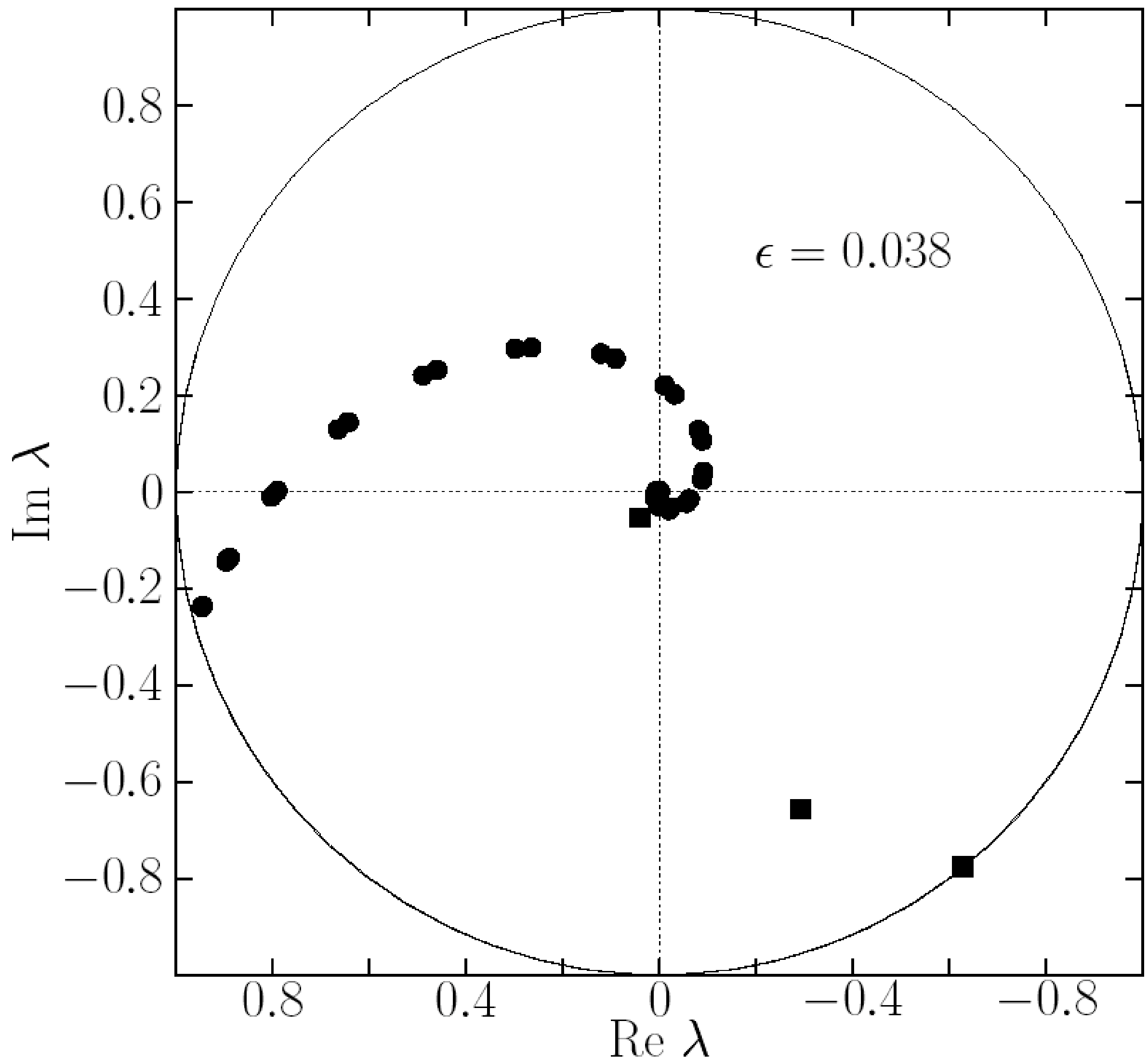
- Definition: Probability for atom in laser field to ionize *decreases* as laser intensity is *increased*.
- Evidence
  - High-frequency approximation (Gavrila, et. al.)
  - Simulations at high frequency (Eberly, et. al.)
  - Experiments at high frequency (FOM, Rice)
- Classical vs. Non-classical: classical dynamics may or may not exhibit stabilization

Open, Time-Periodic Quantum Systems

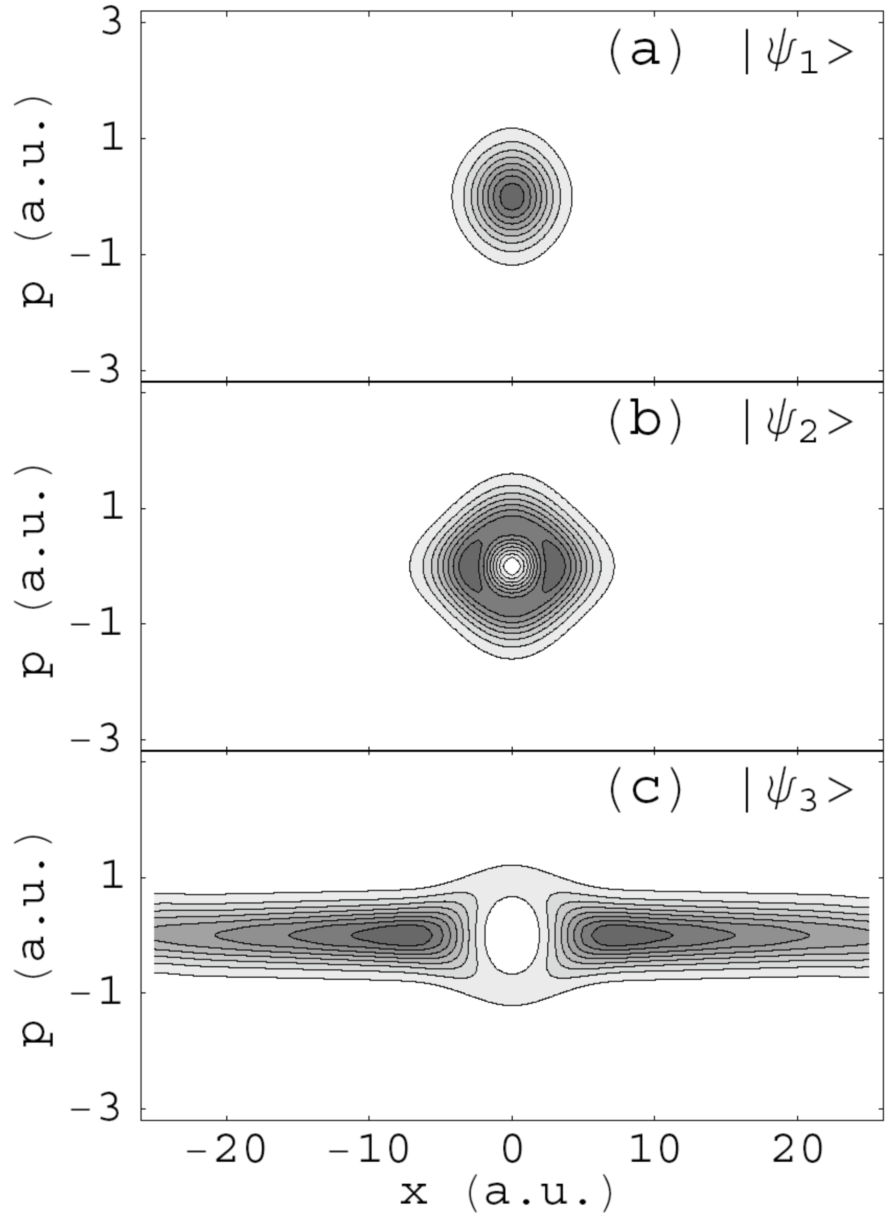
- Floquet Theory
  - Because of the time-periodic driving field this system has no energy eigenstates.
  - Instead we characterize the system using the eigenstates of the one-period time-evolution operator, called the Floquet states.
  - Eigenvalues are of the form e^{-iqT}, where q is the *quasienergy* and $T = 2\pi/\omega$ is the period of the driving field.

- Complex Coordinate Scaling
  - Since the system is open, it can ionize.
  - We can account for this by scaling the *x*-coordinate by a complex phase factor.
  - This makes the Hamiltonian non-Hermitian and the quasienergies complex (q = □ -i□/2, where □ is the photodetachment rate and □=1/□ is the lifetime of the state).
  - We are interested in the *resonance states* (localized, metastable Floquet states).
  - To get accurate *wavefunctions* we must use *exterior* complex coordinate scaling.

- Identifying Resonance States
  - Floquet eigenvalues of continuum states form a spiral.
  - Eigenvalues of resonance states lie off the spiral.

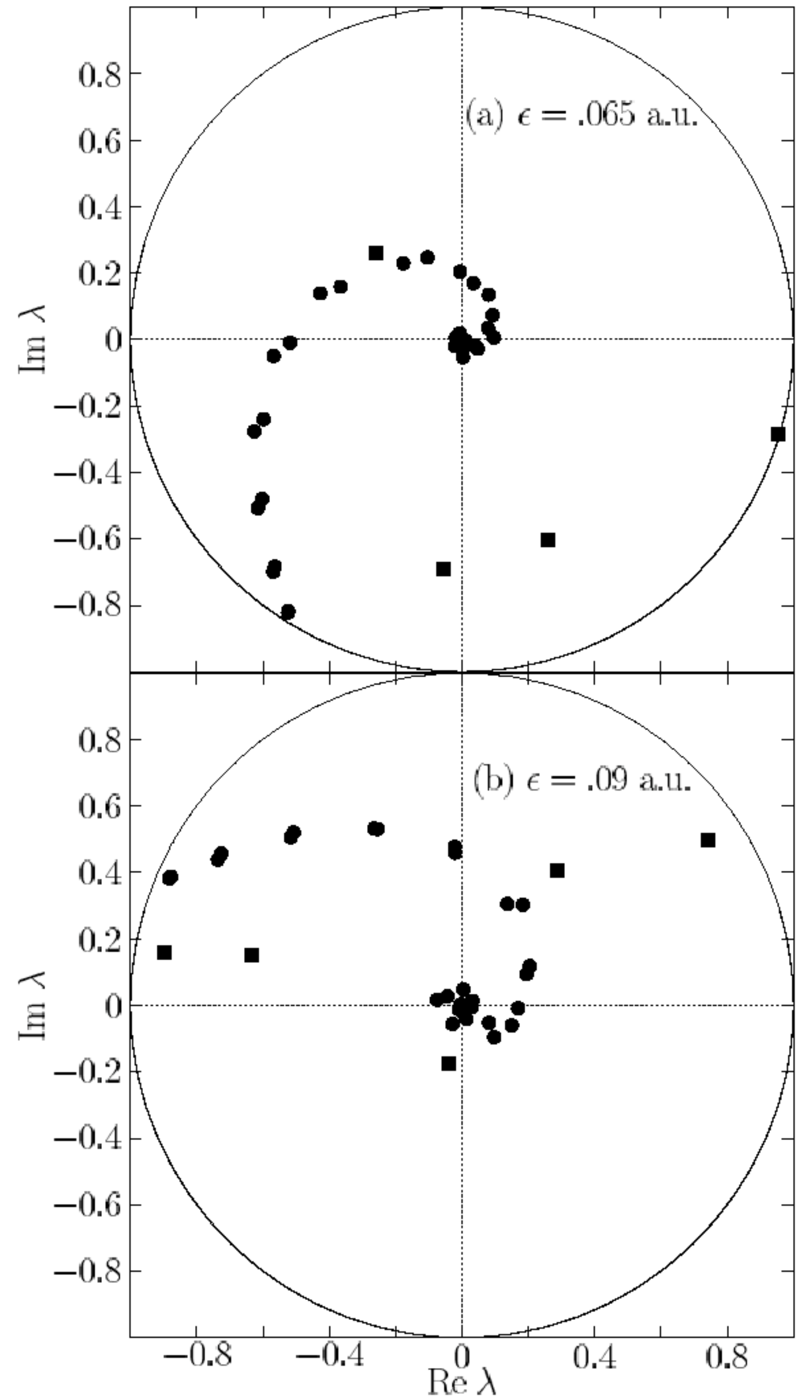


- Husimi Distributions
  - We would like to have a phase-space picture of these resonance states to make comparisons to the classical phase space.
  - Husimi distributions are quasi-probability distributions in phase-space that conform to the uncertainty principle.

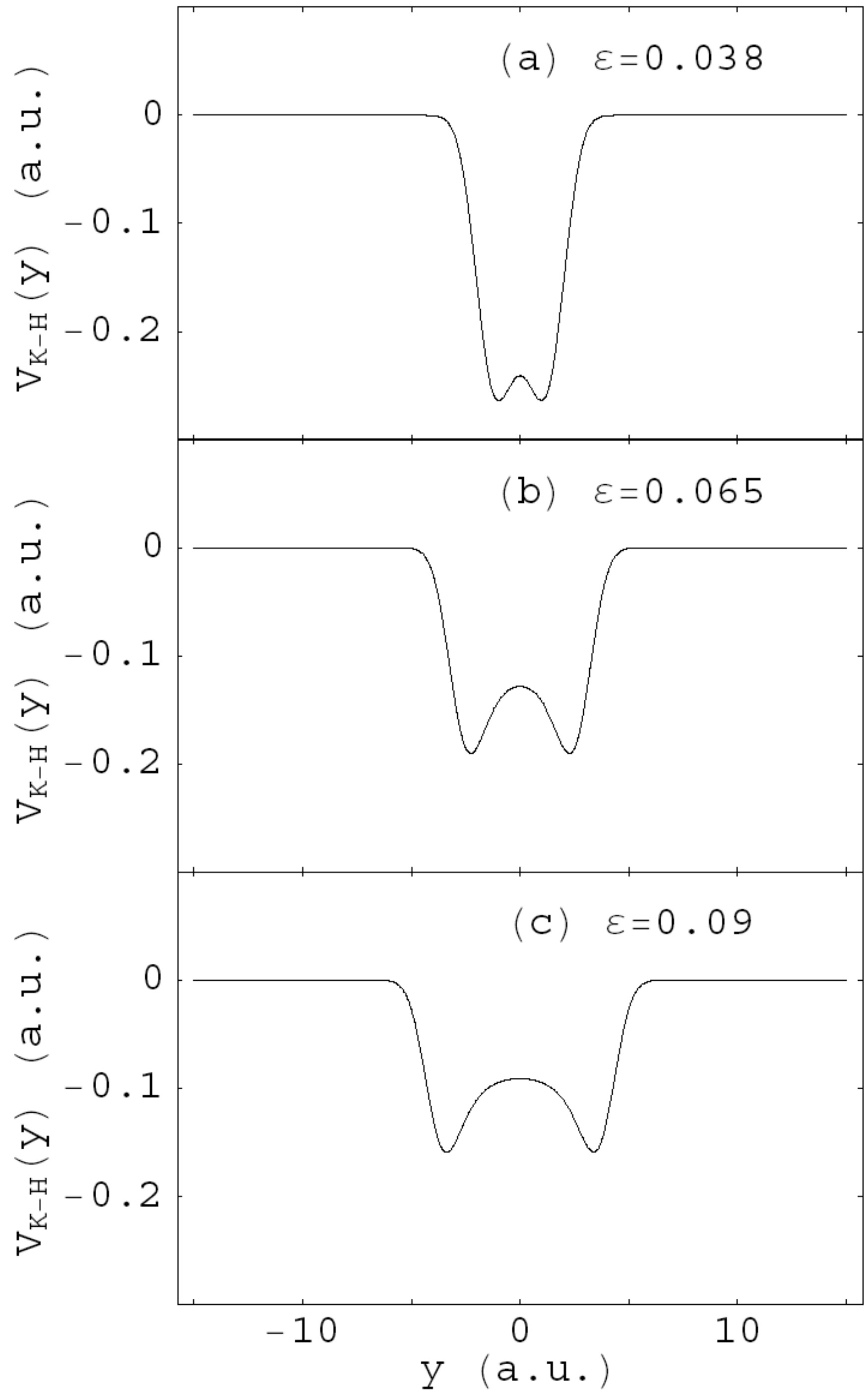


Creation of Resonance States

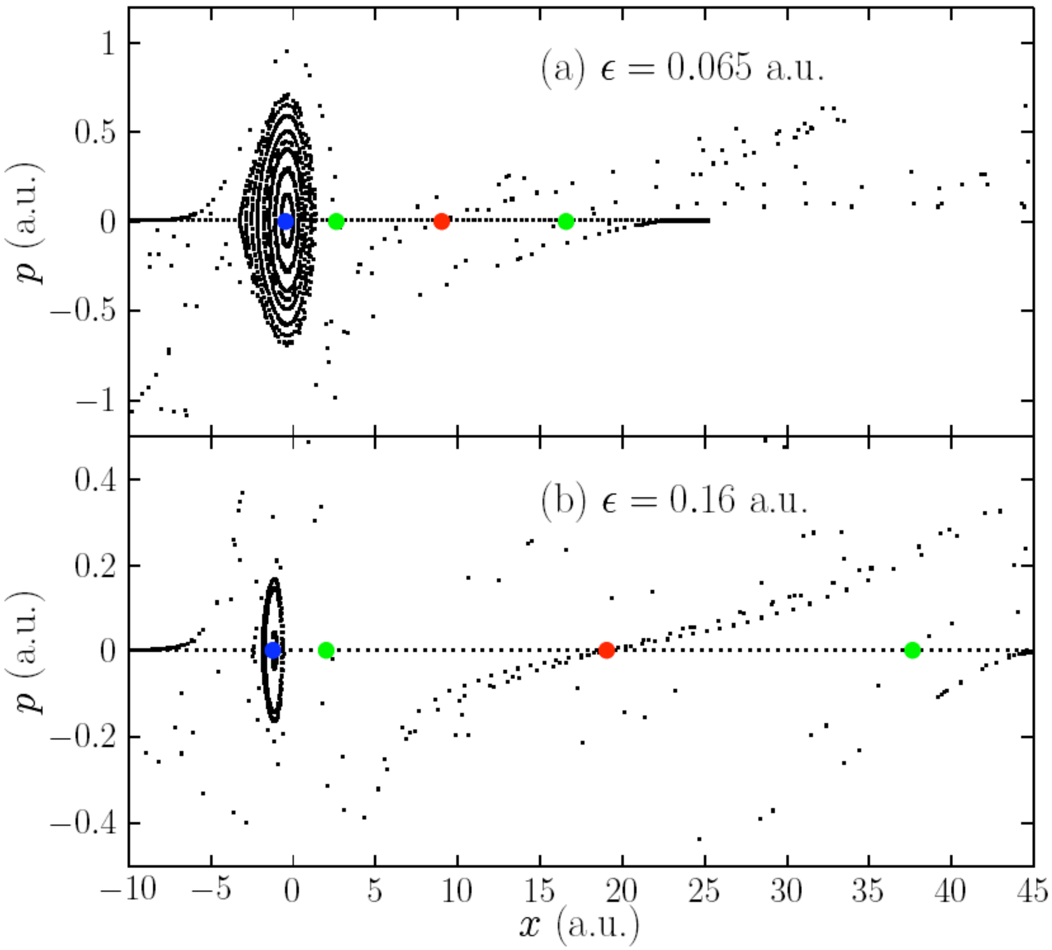
- Results
  - Number of resonance states increases from 3 to 5 as ε is increased from 0.038 to 0.09 a.u.
  - New states are light-induced states.
  - This is surprising because you would expect resonance states to be destroyed as ε is increased.

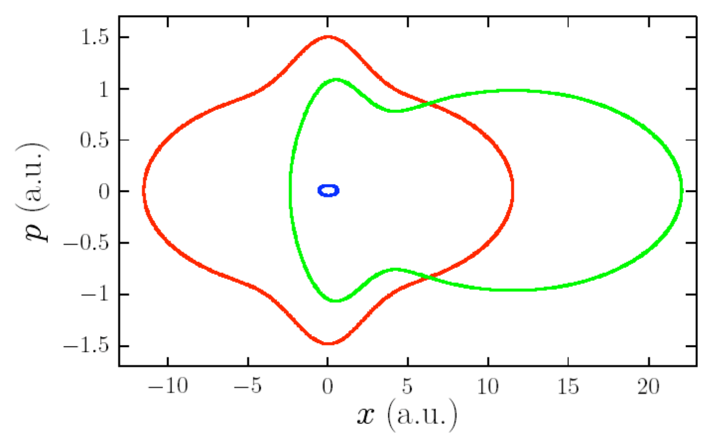


- Original Explanation by Ben-Tal, Moiseyev, and Kosloff, *J. Chem. Phys.* **98**, 9610 (1993).
  - In the Kramers-Henneberger frame (frame of particle that oscillates with driving field) the potential well oscillates back and forth.
  - At high frequencies this oscillating potential can be approximated by the time-averaged potential.
  - Time-averaged K-H potential gets wider and admits more bound states as □is increased.

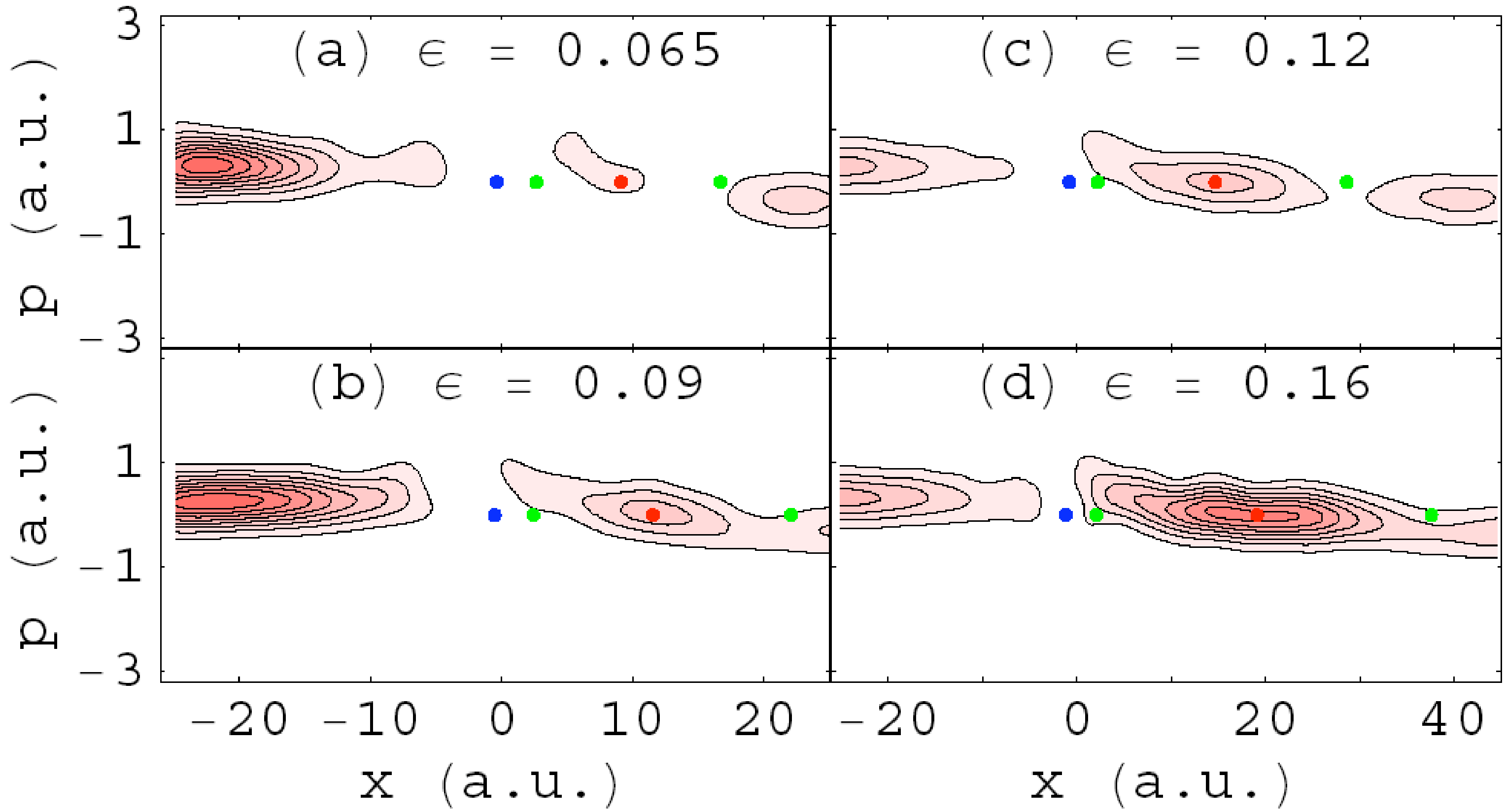


- Classical Motion
  - Stable island near origin surrounded by sea of chaotic ionizing trajectories.
  - Stable island gets smaller as □is increased.
  - Shows that time-averaged K-H potential is not a valid approximation for this low frequency.
  - Periodic orbits A, B, C, and D, with period T.



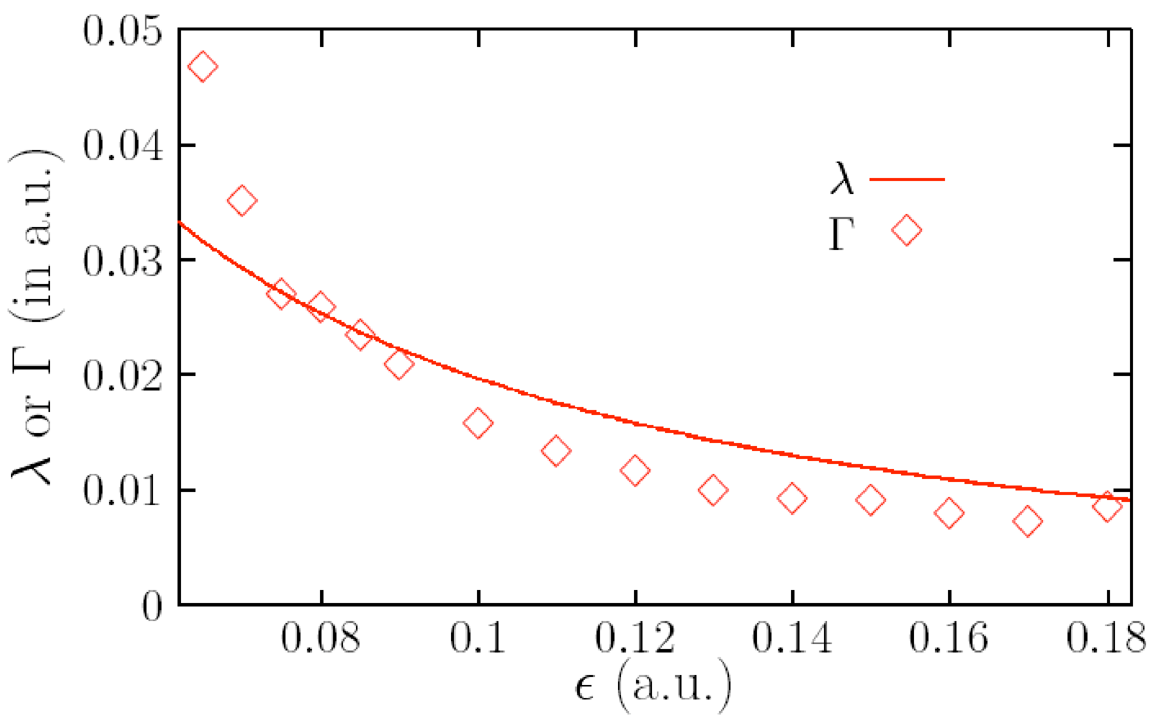


- Scarred Resonance State
  - One of the light-induced resonance states that is created as □is increased has Husimi distributions that are peaked on Orbit C.
  - This state is a scar of Orbit C. It can only be created once the Orbit C has moved sufficiently far from the stable island.



□ and Lyapunov Exponent

- The continuous line shows the Lyapunov exponent (□) of Orbit C, which measures rate at which nearby trajectories move away.
- The data points show the photodetachment rate (□) of the scarred state.
- Both decrease over a similar range of values as □ is increased.
- For □ > 0.13 a.u. the scarred state has the longest lifetime.

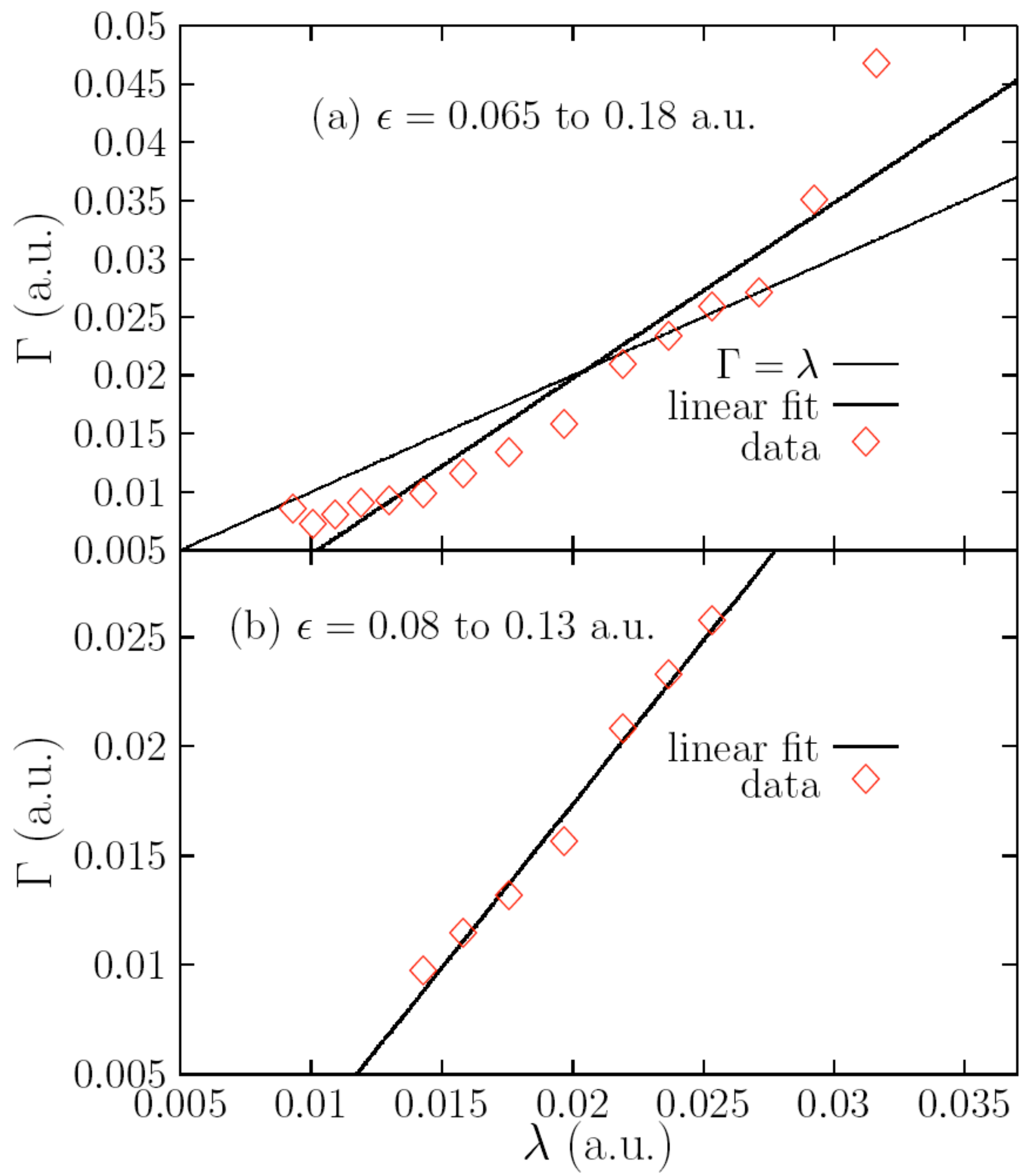


- Correlation of □ and □
  - Strong correlation over all field strengths (R = 0.953).
  - Best-fit line:

□ = 1.505□ □ 0.010 a.u.

  - Very linear in restricted range with R = 0.993.
  - Best-fit line:

□ = 1.496□ □ 0.013 a.u.



Quantum-Classical Correspondence in Atomic Stabilization

- At high frequencies, both classical and quantum systems exhibit stabilization.
- At low frequencies classical system may not exhibit stabilization, but quantum system can.
  - Light-induced states may be scarred on unstable periodic orbits. These states have photodetachment rates that are correlated with the Lyapunov exponent of the unstable periodic orbit.
  - If the Lyapunov exponent decreases with □, then the scarred state will exhibit stabilization.
  - If the scarred state has a photodetachment rate that is small relative to other resonance states, its behavior may dictate that of the system.

Summary

- Light-induced states may be created even as the classical dynamics becomes increasingly unstable.
- These light-induced states may be scarred on unstable periodic orbits.
- The photodetachment rate of a scarred state is correlated to the Lyapunov exponent of the periodic orbit on which it is scarred.
- These scarred, light-induced states may play an important role in non-classical, low-frequency atomic stabilization.